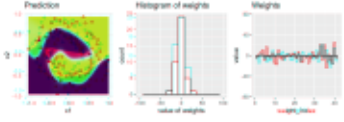
Introduction to Machine Learning

Regularization
Non-Linear Models and Structural Risk Minimization

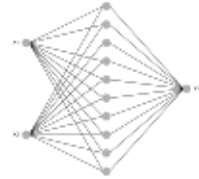


Learning goals

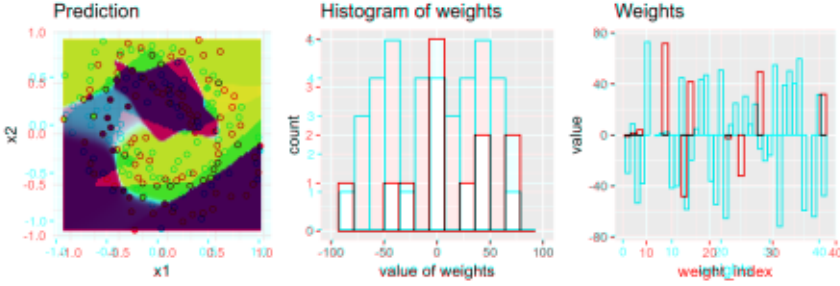
- Regularization even more important in non-linear models
- Norm penalties applied similarly
- Structural risk minimization

REGULARIZATION IN NONLINEAR MODELS

Classification for `spirals` data.
NN with single hidden layer, size 10, $L2$ penalty:



$\lambda = 0$



λ affects smoothness of decision boundary and magnitude of weights

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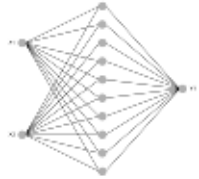
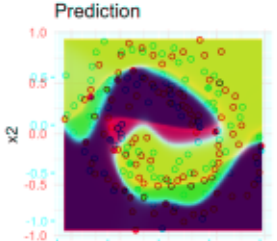
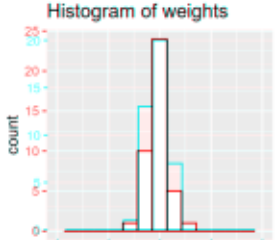
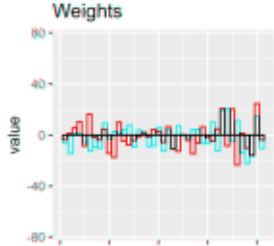


$\lambda = 0.001$







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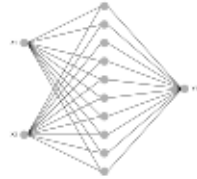
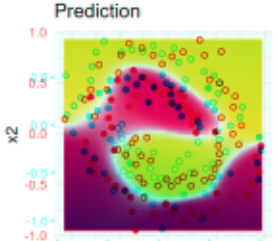
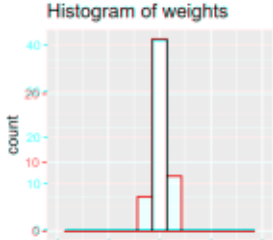
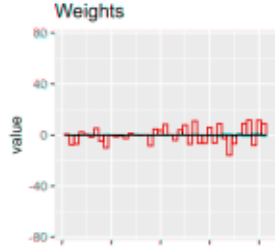


$\lambda = 0.005$







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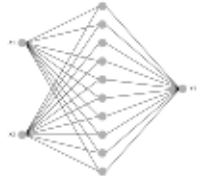

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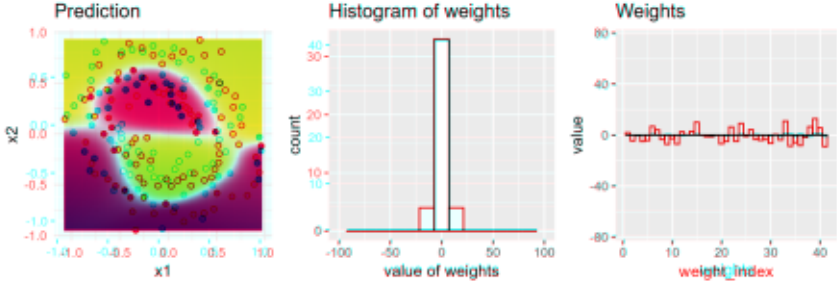

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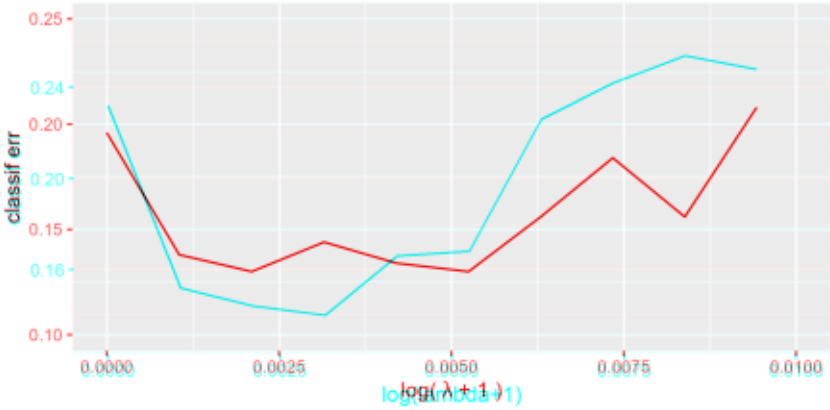
REGULARIZATION IN NONLINEAR MODELS

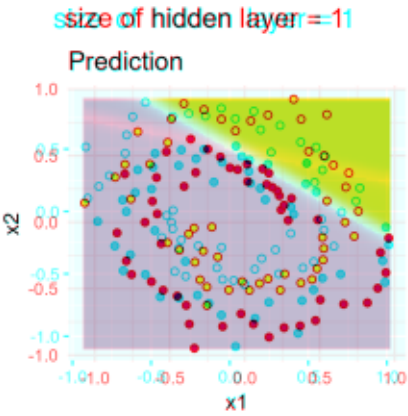
Prevention of overfitting can also be seen in CV.
Same settings as before, but each λ is evaluated with 5x10 REP-CV

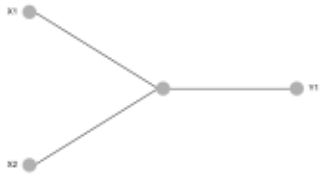


Typical U-shape with sweet spot between overfitting and underfitting

STRUCTURAL RISK MINIMIZATION

Again `spirals`.
NN with 1 hidden layer, and fixed (small) L2 penalty.





Size affects complexity and smoothness of decision boundary

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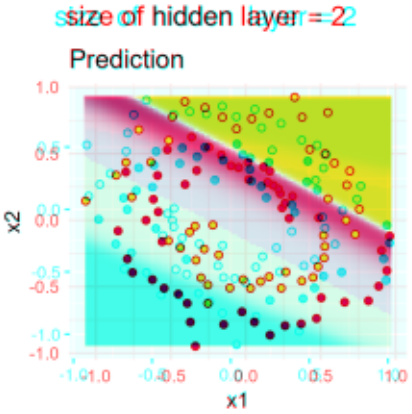

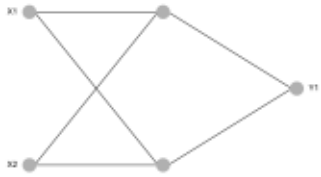

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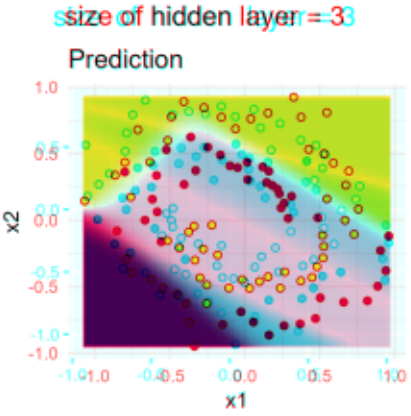

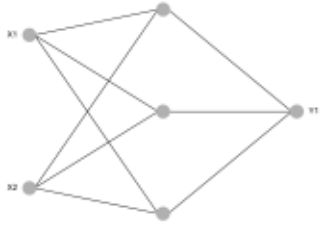

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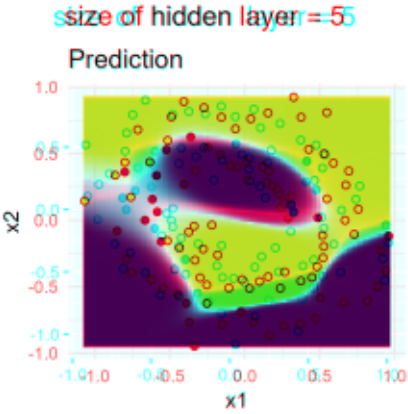

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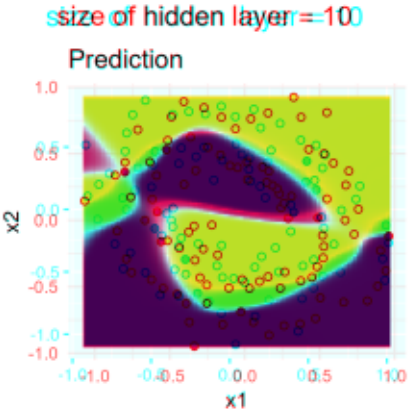

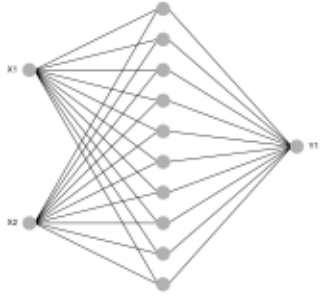

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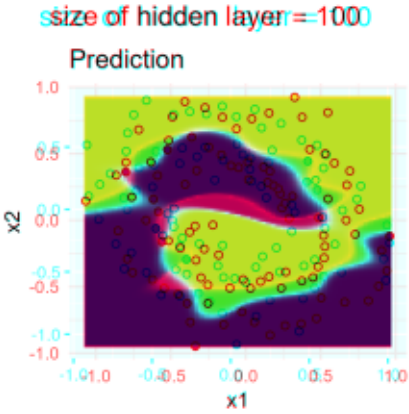
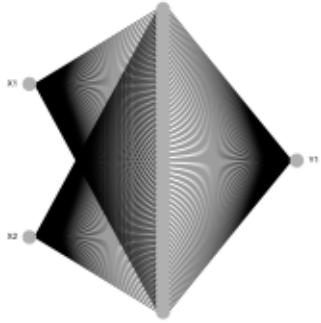
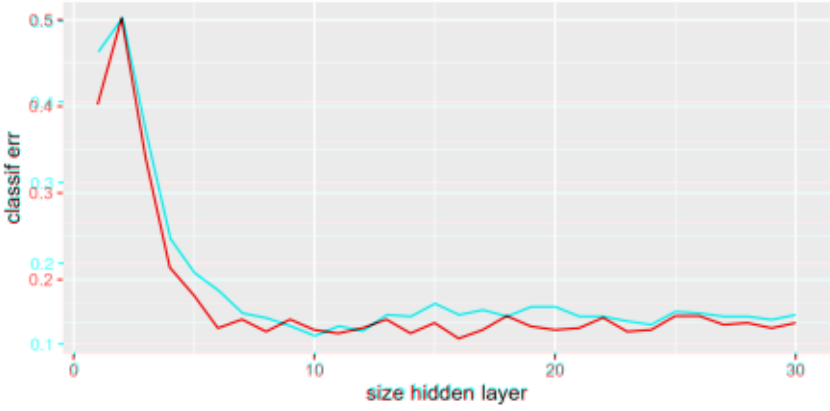




Size affects complexity and smoothness of decision boundary

STRUCTURAL RISK MINIMIZATION

Again, complexity vs CV score.



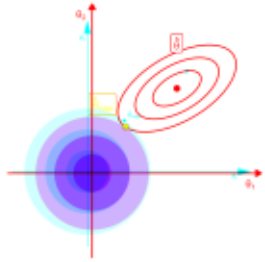
Minimal model with good generalization seems to size=10

STRUCTURAL RISK MINIMIZATION AND RRM

RRM can also be interpreted through SRM,
if we rewrite it in constrained form:

$$\min_{\theta} \quad \sum_{i=1}^{n} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)} \mid \theta\right)\right)$$

$$\text{s.t.} \quad \|\theta\|_2^2 \leq t$$



Can interpret going through λ from large to small as through t from small to large. Constructs series of ERM problems with hypothesis spaces $\mathcal{H}_\lambda$, where we constrain norm of θ to unit balls of growing sizes.