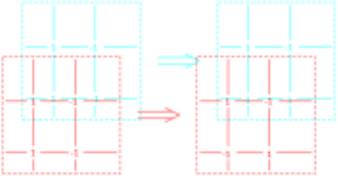
Introduction to Machine Learning

Multiclass Classification

Designing Codebooks and ECOC



Learning goals

- Know what a codebook is
- Understand that codebooks generalize one-vs-one and one-vs-rest
- Know how to define a good codebook and error-correcting output codes (ECOC)
- Know how randomized hill-climbing algorithm is used to find good codebooks

Designing Codebooks and ECOC
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CODEBOOKS

- We have already seen that we can write down principles like one-vs-rest and one-vs-one reduction compactly by so-called **codebooks**.
- During training, a scoring classifier is trained for each column.
- The k-th row is called **codeword** for class k.
- Knowing the principle of **codebooks**, we can define multiclass-to-binary reductions quite flexibly.
- We can now ask ourselves, how to create optimal codebooks.

Class	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$
1	1	-1	-1
2	-1	1	-1
3	- 1	-1	1

Class	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$
1	1	-1	0
2	-1	0	1
3	0	1	-1

Left: one-vs-rest codebook. Right: one-vs-one codebook.

CODEBOOKS: DECIDING LABELS

For a general codebook, once we trained the classifiers, how to predict the class $\hat{y}$ for a new input $\mathbf{x}$?

- When a new sample $\mathbf{x}$ is going to be classified, all classifiers f_k are applied to $\mathbf{x}$, scores are potentially transformed and turned into binary labels by $\text{sgn}(f_k(\mathbf{x}))$.

Class	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$
1	1	1	0
2	-1	1	1
3	0	-1	-1
$\text{sgn}(\hat{f}(\mathbf{x}))$	**-1**	**1**	**-1**

- We obtain a code for the observation $\mathbf{x}$ for which we can calculate the distance to the codewords of the other classes. This can be done by **Hamming distance** (counting the number of bits that differ) or by $L1$-distance.

CODEBOOKS: DECIDING LABELS / 2

- For example, the $L1$-distance between $\text{sgn}\left(\hat{f}(\mathbf{x})\right) = (-1, 1, -1)$ and the class 1 codeword $(1, 1, 0)$ is 3.
- We can do so for all the classes to obtain respective distances:

Classes	Dist
1	3
2	2
3	3

The distance for class 2 is minimal, therefore we predict class 2 for the input $\mathbf{x}$.

ERROR-CORRECTING CODES (ECOC) / 2

Another desirable property is **column separation**:

- Columns should be uncorrelated.
- If two columns k and l are similar or identical, a learning algorithm will make similar (correlated) mistakes in learning f_k and f_l.
- Error-correcting codes only succeed if the errors made in the individual classifiers are relatively uncorrelated, so that the number of simultaneous errors in many classifiers is small.
- Errors in classifiers f_k and f_l will also be highly correlated if the bits in those columns are complementary.
- Try to ensure that columns are neither identical nor complementary.

$\rightarrow$ **We want to maximize distances between rows, and want the distances between columns to not be too small (identical columns) or too high (complementary columns).**

ERROR-CORRECTING CODES (ECOC) / 3

Remark:

- In general, if there are k classes, there will be at most $2^{k-1} - 1$ usable binary columns.
- For example for $k = 3$, there are only $2^3 = 8$ possible columns. Of these, half are complements of the other half. The columns that only contain 1s or the one that only contains $-1s$ are also not usable.

Class	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$	$f_4(\mathbf{x})$	$f_5(\mathbf{x})$	$f_6(\mathbf{x})$	$f_7(\mathbf{x})$	$f_8(\mathbf{x})$
1	-1	-1	-1	-1	1	1	1	1
2	-1	- 1	1	1	-1	-1	1	1
3	-1	1	-1	1	-1	1	-1	1

ERROR-CORRECTING CODES (ECOC) / 4

Assume we have the budget to train L binary classifiers and now want to find an error-correcting code with maximal row and column separation.

- For only few classes $g \leq 11$, exhaustive search can be performed and a codebook that has good row and column separation is chosen.
- However, for many classes $g > 11$, it becomes more and more challenging to find the optimal codebook with codewords of length L.
- *Dietterich et al.* employed a randomized hill-climbing algorithm for this task.