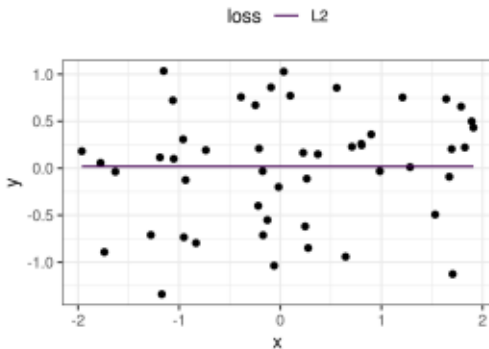


L2-LOSS: OPTIMAL CONSTANT MODEL / 2

The optimizer \hat{f}_c of the empirical risk is \bar{y} (the empirical mean over $y^{(i)}$), which is the empirical estimate for $\mathbb{E}_y [y]$.



L2-LOSS: OPTIMAL CONSTANT MODEL / 3

Proof:

For the optimal constant model f_c^* for the L2-loss $L(y, f) = (y - f)^2$ we solve the optimization problem

$$\arg \min_{f \in \mathcal{H}} \mathcal{R}_{\text{emp}}(f) = \arg \min_{\theta \in \mathbb{R}} \sum_{i=1}^n (y^{(i)} - \theta)^2.$$

We calculate the first derivative of \mathcal{R}_{emp} w.r.t. θ and set it to 0:

$$\begin{aligned} \frac{\partial \mathcal{R}_{\text{emp}}(\theta)}{\partial \theta} &= -2 \sum_{i=1}^n (y^{(i)} - \theta) \stackrel{!}{=} 0 \\ \sum_{i=1}^n y^{(i)} - n\theta &= 0 \\ \hat{\theta} &= \frac{1}{n} \sum_{i=1}^n y^{(i)} =: \bar{y}. \end{aligned}$$

