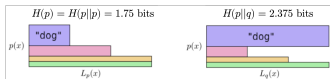
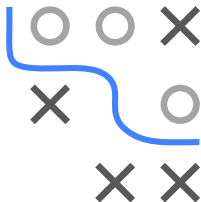


# Introduction to Machine Learning

## Information Theory

## Source Coding and Cross-Entropy

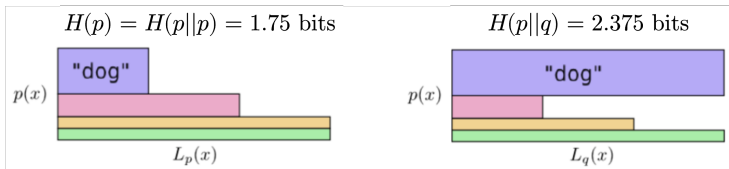
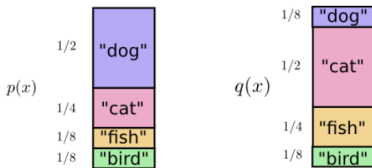
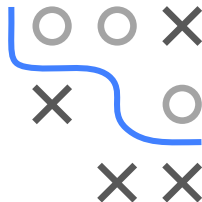


### Learning goals

- Know connection between source coding and (cross-)entropy
- Know that the entropy of the source distribution is the lower bound for the average code length

# SOURCE CODING AND CROSS-ENTROPY

- For a random source / distribution  $p$ , the minimal number of bits to optimally encode messages from is the entropy  $H(p)$ .
- If the optimal code for a different distribution  $q(x)$  is instead used to encode messages from  $p(x)$ , expected code length will grow.

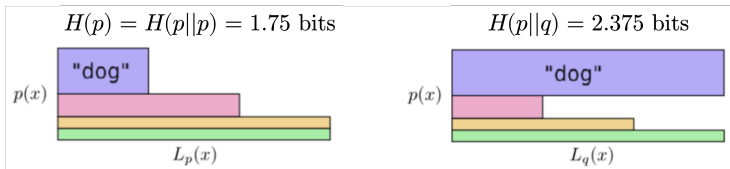
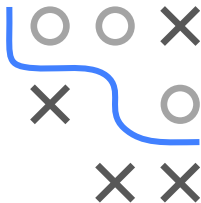


**Figure:**  $L_p(x)$ ,  $L_q(x)$  are the optimal code lengths for  $p(x)$  and  $q(x)$

# SOURCE CODING AND CROSS-ENTROPY / 2

**Cross-entropy** is the average length of communicating an event from one distribution with the optimal code for another distribution (assume they have the same domain  $\mathcal{X}$  as in KL).

$$H(p||q) = \sum_{x \in \mathcal{X}} p(x) \log \left( \frac{1}{q(x)} \right) = - \sum_{x \in \mathcal{X}} p(x) \log (q(x))$$

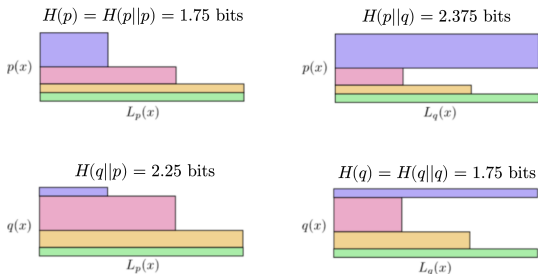


**Figure:**  $L_p(x)$ ,  $L_q(x)$  are the optimal code lengths for  $p(x)$  and  $q(x)$

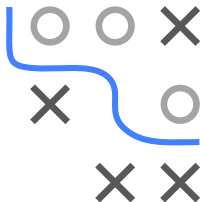
We directly see: cross-entropy of  $p$  with itself is entropy:

$$H(p||p) = H(p).$$

# SOURCE CODING AND CROSS-ENTROPY / 3

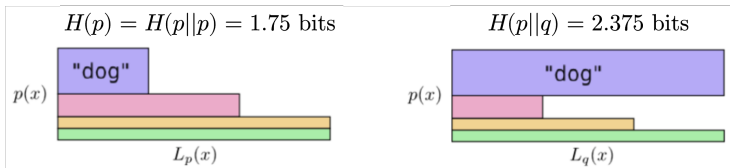


Credit: Chris Olah



- In top,  $H(p||q)$  is greater than  $H(p)$  primarily because the blue event that is very likely under  $p$  has a very long codeword in  $q$ .
- Same, in bottom, for pink when we go from  $q$  to  $p$ .
- Note that  $H(p||q) \neq H(q||p)$ .

# SOURCE CODING AND CROSS-ENTROPY / 4



**Figure:**  $L_p(x)$ ,  $L_q(x)$  are the optimal code lengths for  $p(x)$  and  $q(x)$

- Let  $x'$  denote the symbol "dog". The difference in code lengths is:

$$\log\left(\frac{1}{q(x')}\right) - \log\left(\frac{1}{p(x')}\right) = \log\frac{p(x')}{q(x')}$$

- If  $p(x') > q(x')$ , this is positive, if  $p(x') < q(x')$ , it is negative.
- The expected difference is KL, if we encode symbols from  $p$ :

$$D_{KL}(p||q) = \sum_{x \in \mathcal{X}} p(x) \cdot \log\frac{p(x)}{q(x)}$$

