# **Optimization in Machine Learning**

# Algorithms for linear programs

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#### Learning goals

- Definition
- Max. Likelihood
- Normal regression
- Risk Minimization

The Simplex algorithm is the most important method for solving Linear programming. It was published in 1947 by Georg Dantzig.

**Basic idea:** start from an arbitrary corner of the polytope. Run along this edge as long as the solution improves. Find a new edge, ...

**Output:** a path along the corners of the polytope that ends at the optimal point of the polytope.

Since linear programming is a **convex** optimization problem, the optimal corner found in this way is also a global optimum.

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The simplex algorithm can be divided into two steps:

- Phase I: determination of a starting point
- Phase II: determination of the optimal solution

To be able to start, a starting point must first be found in **Phase I**, i.e. a feasible corner  $x_0$ .

In **phase II** this solution is iteratively improved by searching for an edge that improves the solution and running along it to the next corner.



Phase I:

One way to find a starting point  $x_0$  is to solve a auxiliary linear problem with artificial variables  $\epsilon$ :

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$$\begin{array}{ll} \min_{\epsilon_1,...,\epsilon_m} & \sum_{i=1}^m \epsilon_i \\ \text{s.t.} & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{\epsilon} \geq \boldsymbol{b} \\ & \epsilon_1,...,\epsilon_m \geq \boldsymbol{0} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{array}$$

• A feasible starting point for the auxiliary problem is  $\mathbf{x} = \mathbf{0}$  and  $\epsilon_i = \begin{cases} 0 & \text{if } b_i < 0 \\ b_i & \text{if } b_i \ge 0 \end{cases}$ 

- We then apply phase II of the simplex algorithm to the auxiliary problem.
- If the original problem has a feasible solution, then the optimal solution of the auxiliary problem **must** be  $\epsilon = (0, ..., 0)$  (all artificial variables disappear) and the objective function is 0.
- If we find a solution with  $\epsilon = 0$ , then we have found a valid starting point.
- If we do not find a solution with  $\epsilon = \mathbf{0}$ , the problem can not be solved.

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#### Example:

$$\begin{array}{ll}
\min_{\mathbf{x}\in\mathbb{R}^2} & -x_1 - x_2 \\
\text{s.t.} & x_1 - x_2 \ge -0.5 \\
& -x_1 - 2x_2 \ge -2 \\
& -2x_1 - x_2 \ge -2 \\
& -x_1 + x_2 \ge -0.5 \\
& \mathbf{x} \ge 0
\end{array}$$

A starting point is the corner (0, 0).

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