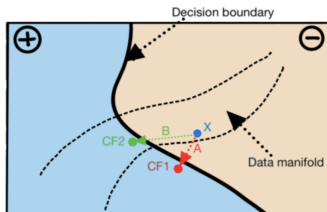
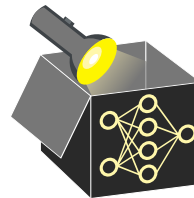


Interpretable Machine Learning

Counterfactual Explanations

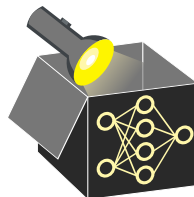
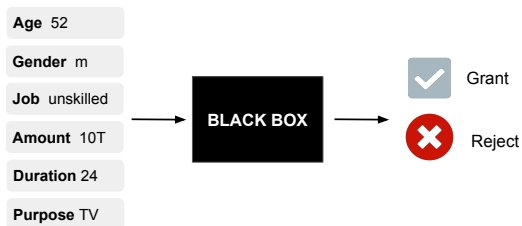


Learning goals

- Understand the motivation behind CEs
- See the mathematical foundation of CEs

EXAMPLE: CREDIT RISK APPLICATION

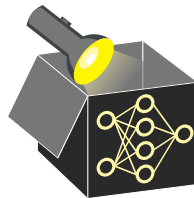
- x : customer and credit information
- y : grant or reject credit



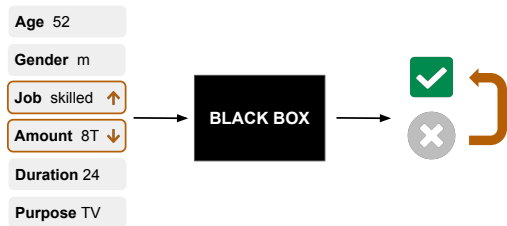
Questions:

- Why was the credit rejected?
- Is it a fair decision?
- **How should x be changed so that the credit is accepted?**

EXAMPLE: CREDIT RISK APPLICATION



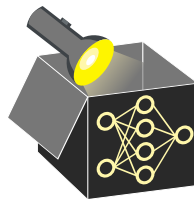
Counterfactual Explanations provide answers in the form of "What-If"-scenarios.



"If the person was more skilled and the credit amount had been reduced to \$8.000, the credit would have been granted."

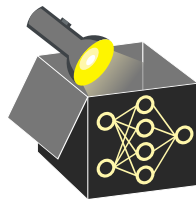
COUNTERFACTUAL EXPLANATIONS: MAIN IDEA

- Counterfactual explanations == counterfactuals == CEs



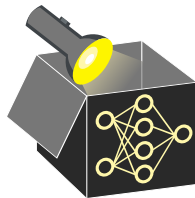
COUNTERFACTUAL EXPLANATIONS: MAIN IDEA

- Counterfactual explanations == counterfactuals == CEs
- Explain particular predictions of an ML model by presenting an alternative input whose prediction equals a desired outcome

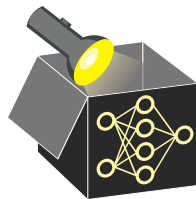


COUNTERFACTUAL EXPLANATIONS: MAIN IDEA

- Counterfactual explanations == counterfactuals == CEs
- Explain particular predictions of an ML model by presenting an alternative input whose prediction equals a desired outcome
- Represent **close neighbors** of a data point we are interested in, but belonging to the **desired outcome**

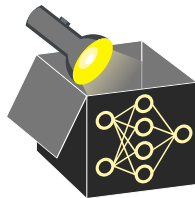


COUNTERFACTUAL EXPLANATIONS: MAIN IDEA



- Counterfactual explanations == counterfactuals == CEs
- Explain particular predictions of an ML model by presenting an alternative input whose prediction equals a desired outcome
- Represent **close neighbors** of a data point we are interested in, but belonging to the **desired outcome**
- Reveal which minimal changes to the input are sufficient to receive a different outcome
~> Useful if there is a chance to change the input features (e.g., by changing behaviour)

COUNTERFACTUAL EXPLANATIONS: MAIN IDEA

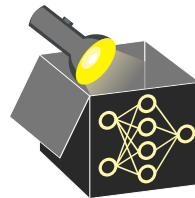


- Counterfactual explanations == counterfactuals == CEs
- Explain particular predictions of an ML model by presenting an alternative input whose prediction equals a desired outcome
- Represent **close neighbors** of a data point we are interested in, but belonging to the **desired outcome**
- Reveal which minimal changes to the input are sufficient to receive a different outcome
~> Useful if there is a chance to change the input features (e.g., by changing behaviour)
- The targeted audience of CEs are often end-users

AIMS & ROLES

CEs can serve various purposes; the user can decide what to learn from them. For example:

“If the person had been **one year older** and the **credit amount had been increased** to \$12.000, the credit would have been granted.”



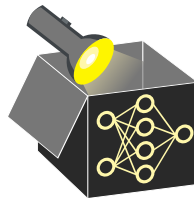
AIMS & ROLES

CEs can serve various purposes; the user can decide what to learn from them. For example:

“If the person had been **one year older** and the **credit amount had been increased** to \$12.000, the credit would have been granted.”

- **Guidance for future actions:**

Ok, I will apply again next year for the higher amount.



AIMS & ROLES

CEs can serve various purposes; the user can decide what to learn from them. For example:

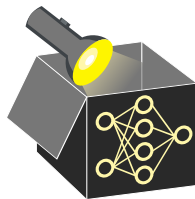
"If the person had been **one year older** and the **credit amount had been increased** to \$12.000, the credit would have been granted."

- **Guidance for future actions:**

Ok, I will apply again next year for the higher amount.

- **Provide reasons:**

Interesting, I did not know that age plays a role in loan applications.



AIMS & ROLES

CEs can serve various purposes; the user can decide what to learn from them. For example:

"If the person had been **one year older** and the **credit amount had been increased** to \$12.000, the credit would have been granted."

- **Guidance for future actions:**

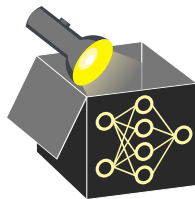
Ok, I will apply again next year for the higher amount.

- **Provide reasons:**

Interesting, I did not know that age plays a role in loan applications.

- **Provide grounds to contest the decision:**

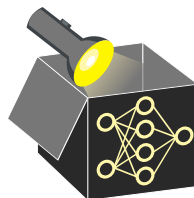
How dare you, I do not want to be discriminated for my age in an application.



AIMS & ROLES

CEs can serve various purposes; the user can decide what to learn from them. For example:

"If the person had been **one year older** and the **credit amount had been increased** to \$12.000, the credit would have been granted."



- **Guidance for future actions:**

Ok, I will apply again next year for the higher amount.

- **Provide reasons:**

Interesting, I did not know that age plays a role in loan applications.

- **Provide grounds to contest the decision:**

How dare you, I do not want to be discriminated for my age in an application.

- **Detect model biases:**

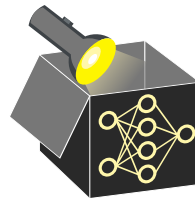
There is a bug, an increase in amount should not increase approval rates.

PHILOSOPHICAL BASIS

Counterfactuals have a long-standing tradition in analytic philosophy

↪ According to ▶ Lewis (1973), a **counterfactual conditional** is a statement of the form:

"If S was the case, Q would have been the case."



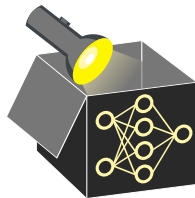
PHILOSOPHICAL BASIS

Counterfactuals have a long-standing tradition in analytic philosophy

~> According to ▶ Lewis (1973), a **counterfactual conditional** is a statement of the form:

"If S was the case, Q would have been the case."

- S is an event that must relate to a past event that didn't occur
~> counterfactuals run **contrary** to the **facts**



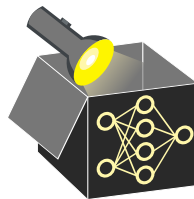
PHILOSOPHICAL BASIS

Counterfactuals have a long-standing tradition in analytic philosophy

↪ According to ▶ Lewis (1973), a **counterfactual conditional** is a statement of the form:

“If S was the case, Q would have been the case.”

- S is an event that must relate to a past event that didn't occur
↪ counterfactuals run **contrary** to the **facts**
- Above statement is true, if in all possible worlds most similar to the actual world where S had been the case, Q would have been the case



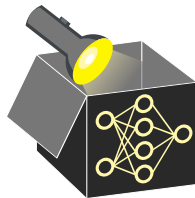
PHILOSOPHICAL BASIS

Counterfactuals have a long-standing tradition in analytic philosophy

↪ According to ▶ Lewis (1973), a **counterfactual conditional** is a statement of the form:

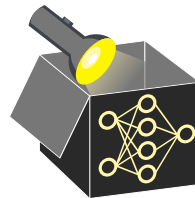
“If S was the case, Q would have been the case.”

- S is an event that must relate to a past event that didn't occur
↪ counterfactuals run **contrary** to the **facts**
- Above statement is true, if in all possible worlds most similar to the actual world where S had been the case, Q would have been the case
- A world is similar to another if laws are maximally preserved between the worlds and only a few facts are changed



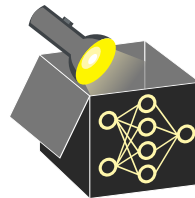
PHILOSOPHICAL BASIS

- Counterfactuals have largely been studied to explain causal dependence



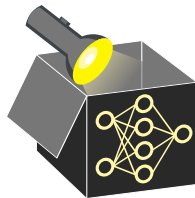
PHILOSOPHICAL BASIS

- Counterfactuals have largely been studied to explain causal dependence
- Causal dependence underlies the explanatory power
 - ~> good CEs point to critical causal factors that drove the algorithmic decision



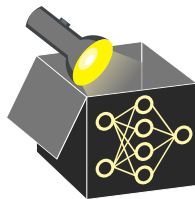
PHILOSOPHICAL BASIS

- Counterfactuals have largely been studied to explain causal dependence
- Causal dependence underlies the explanatory power
 - ↪ good CEs point to critical causal factors that drove the algorithmic decision
- If maximal closeness is relaxed, causally irrelevant factors can become part of the explanation
 - ↪ e.g., decreasing loan amount by \$20.000 and being one year older is recommended by the explainer although only loan amount might be causally relevant



PHILOSOPHICAL BASIS

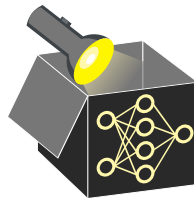
- Counterfactuals have largely been studied to explain causal dependence
- Causal dependence underlies the explanatory power
 - ↪ good CEs point to critical causal factors that drove the algorithmic decision
- If maximal closeness is relaxed, causally irrelevant factors can become part of the explanation
 - ↪ e.g., decreasing loan amount by \$20.000 and being one year older is recommended by the explainer although only loan amount might be causally relevant
- CEs are often contrastive, i.e., they explain a decision by referring to an alternative outcome
 - ↪ e.g., if the loan applicant was 30 instead of 60 years old, the approved loan would have been over \$100.000 instead of \$40.000



MATHEMATICAL PERSPECTIVE

Terminology:

- \mathbf{x} : original/factual datapoint whose prediction we want to explain
- $y' \subset \mathbb{R}^g$: desired prediction ($y' = 1000$ or $y' = \text{"grant credit"}$) or interval ($y' = [1000, \infty[$)



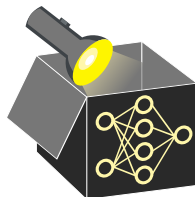
MATHEMATICAL PERSPECTIVE

Terminology:

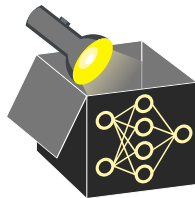
- \mathbf{x} : original/factual datapoint whose prediction we want to explain
- $y' \subset \mathbb{R}^g$: desired prediction ($y' = 1000$ or $y' = \text{"grant credit"}$) or interval ($y' = [1000, \infty[$)

A **valid** counterfactual \mathbf{x}' is a datapoint:

- 1 whose prediction $\hat{f}(\mathbf{x}')$ is equal to the desired prediction y'
- 2 that is maximally close to the original datapoint \mathbf{x}



MATHEMATICAL PERSPECTIVE



Terminology:

- \mathbf{x} : original/factual datapoint whose prediction we want to explain
- $y' \in \mathbb{R}^g$: desired prediction ($y' = 1000$ or $y' = \text{"grant credit"}$) or interval ($y' = [1000, \infty[$)

A **valid** counterfactual \mathbf{x}' is a datapoint:

- 1 whose prediction $\hat{f}(\mathbf{x}')$ is equal to the desired prediction y'
- 2 that is maximally close to the original datapoint \mathbf{x}

Reformulate these two objectives (denoted by o_1 and o_2) as optimization problem:

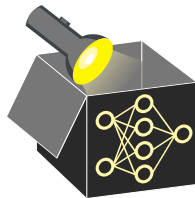
$$\arg \min_{\mathbf{x}'} \lambda_1 o_p(\hat{f}(\mathbf{x}'), y') + \lambda_2 o_f(\mathbf{x}', \mathbf{x})$$

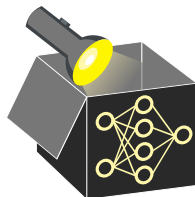
- λ_1 and λ_2 balance the two objectives
- Choice of o_p (distance on prediction space) and of o_f (distance on feature space) is crucial

MATHEMATICAL PERSPECTIVE

► Dandl et al. (2020)

- Regression: o_p could be the L₁-distance $o_p(\hat{f}(\mathbf{x}'), y') = |\hat{f}(\mathbf{x}') - y'|$
- Classification: L₁-distance for scores and 0-1 Loss for labels, e.g.,
 $o_p(\hat{f}(\mathbf{x}'), y') = \mathcal{I}_{\{\hat{f}(\mathbf{x}') \neq y'\}}$





- Regression: o_p could be the L₁-distance $o_p(\hat{f}(\mathbf{x}'), y') = |\hat{f}(\mathbf{x}') - y'|$
- Classification: L₁-distance for scores and 0-1 Loss for labels, e.g., $o_p(\hat{f}(\mathbf{x}'), y') = \mathcal{I}_{\{\hat{f}(\mathbf{x}') \neq y'\}}$
- o_f could be the Gower distance (suitable for mixed feature space):

$$o_f(\mathbf{x}', \mathbf{x}) = d_G(\mathbf{x}', \mathbf{x}) = \frac{1}{p} \sum_{j=1}^p \delta_G(x'_j, x_j) \in [0, 1]$$

The value of δ_G depends on the feature type (numerical or categorical):

$$\delta_G(x'_j, x_j) = \begin{cases} \frac{1}{\widehat{R}_j} |x'_j - x_j| & \text{if } x_j \text{ is numerical} \\ \mathcal{I}_{\{x'_j \neq x_j\}} & \text{if } x_j \text{ is categorical} \end{cases}$$

with \widehat{R}_j as the value range of feature j in the training dataset (to ensure that $\delta_G(x'_j, x_j) \in [0, 1]$)

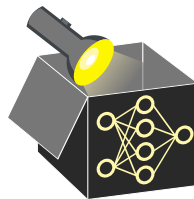
FURTHER OBJECTIVES

Additional constraints can improve the explanation quality of the corresponding CEs

↪ popular constraints include sparsity and plausibility

Sparsity:

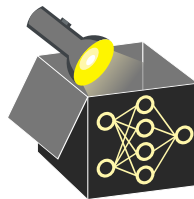
- End-users often prefer short over long explanations
↪ counterfactuals should be **sparse**



FURTHER OBJECTIVES

Additional constraints can improve the explanation quality of the corresponding CEs

↪ popular constraints include sparsity and plausibility



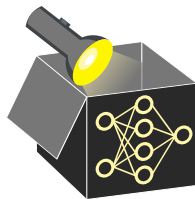
Sparsity:

- End-users often prefer short over long explanations
↪ counterfactuals should be **sparse**
- Objective σ_f can take the number of changed features into account (but does not have to)
↪ e.g., the L_0 - and the L_1 -norm (similar to LASSO) can do this

FURTHER OBJECTIVES

Additional constraints can improve the explanation quality of the corresponding CEs

↪ popular constraints include sparsity and plausibility



Sparsity:

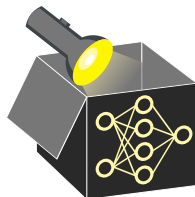
- End-users often prefer short over long explanations
↪ counterfactuals should be **sparse**
- Objective o_f can take the number of changed features into account (but does not have to)
↪ e.g., the L_0 - and the L_1 -norm (similar to LASSO) can do this
- Independently from o_f , sparsity in the changes can be additionally considered by another objective that counts the number of changed features via the L_0 -norm:

$$o_s(\mathbf{x}', \mathbf{x}) = \sum_{j=1}^p \mathcal{I}_{\{x'_j \neq x_j\}}$$

FURTHER OBJECTIVES

Plausibility:

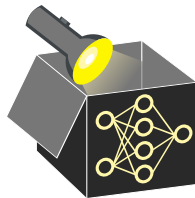
- CEs should suggest plausible alternatives
 - ↪ e.g., not plausible to suggest to raise your income and get unemployed at the same time



FURTHER OBJECTIVES

Plausibility:

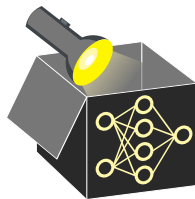
- CEs should suggest plausible alternatives
 - ↪ e.g., not plausible to suggest to raise your income and get unemployed at the same time
- CEs should be realistic and adhere to data manifold or originate from distribution of \mathcal{X}
 - ↪ avoid unrealistic combinations of feature values



FURTHER OBJECTIVES

Plausibility:

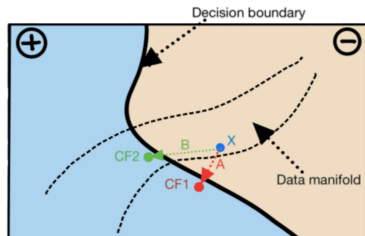
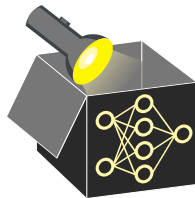
- CEs should suggest plausible alternatives
 - ↪ e.g., not plausible to suggest to raise your income and get unemployed at the same time
- CEs should be realistic and adhere to data manifold or originate from distribution of \mathcal{X}
 - ↪ avoid unrealistic combinations of feature values
- Estimating joint distribution of training data is complex, especially for mixed feature spaces
 - ↪ Proxy: ensure that \mathbf{x}' is close to training data \mathbf{X}



FURTHER OBJECTIVES

Plausibility:

- CEs should suggest plausible alternatives
 - ↪ e.g., not plausible to suggest to raise your income and get unemployed at the same time
- CEs should be realistic and adhere to data manifold or originate from distribution of \mathcal{X}
 - ↪ avoid unrealistic combinations of feature values
- Estimating joint distribution of training data is complex, especially for mixed feature spaces
 - ↪ Proxy: ensure that \mathbf{x}' is close to training data \mathbf{X}



Example from [Verma et al. \(2020\)](#)

- Two possible paths for \mathbf{x} , originally classified to \ominus
- Two valid CEs in class \oplus : **CF1** and **CF2**
- **Path A** for **CF1** is shorter
- **Path B** for **CF2** is longer but adheres to data manifold

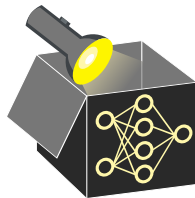
FURTHER OBJECTIVES

To ensure plausibility, o_4 could, e.g., be the Gower distance of \mathbf{x}' to its nearest data point of the training dataset which we denote $\mathbf{x}^{[1]}$:

$$o_4(\mathbf{x}', \mathbf{X}) = d_G(\mathbf{x}', \mathbf{x}^{[1]}) = \frac{1}{p} \sum_{j=1}^p \delta_G(x'_j, x_j^{[1]})$$

We can extend the previous optimization problem by adding o_s (for sparsity) and o_4 (for plausibility):

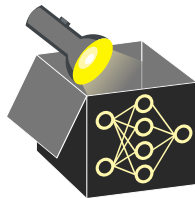
$$\arg \min_{\mathbf{x}'} \lambda_1 o_p(\hat{f}(\mathbf{x}'), y') + \lambda_2 o_f(\mathbf{x}', \mathbf{x}) + \lambda_3 o_s(\mathbf{x}', \mathbf{x}) + \lambda_4 o_4(\mathbf{x}', \mathbf{X})$$



REMARKS: THE RASHOMON EFFECT

Issue (Rashomon effect):

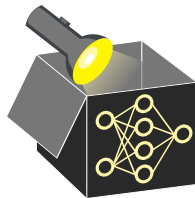
- Solution to the optimization problem might not be unique
- Many equally close CE might exist that obtain the desired prediction
⇒ Many different equally good explanations for the same decision exist



REMARKS: THE RASHOMON EFFECT

Issue (Rashomon effect):

- Solution to the optimization problem might not be unique
- Many equally close CE might exist that obtain the desired prediction
⇒ Many different equally good explanations for the same decision exist



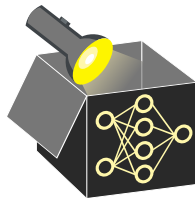
Possible solutions:

- Present all CEs for a given \mathbf{x} (but: time and human processing capacity is limited)
- Focus on one or few CEs (but: by which criterion should they be selected?)

REMARKS: THE RASHOMON EFFECT

Issue (Rashomon effect):

- Solution to the optimization problem might not be unique
- Many equally close CE might exist that obtain the desired prediction
⇒ Many different equally good explanations for the same decision exist



Possible solutions:

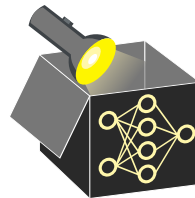
- Present all CEs for a given \mathbf{x} (but: time and human processing capacity is limited)
- Focus on one or few CEs (but: by which criterion should they be selected?)

Note:

- As the model is generally non-linear, inconsistent and diverse CEs can arise e.g. suggesting either an increase or decrease in credit duration (confuses the explainee)
- How to deal with the Rashomon effect is considered an open problem in IML

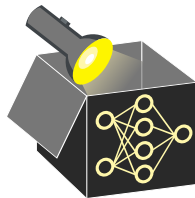
REMARKS: MODEL OR REAL-WORLD

- Most CEs provide explanations of model predictions, but CEs might appear to explain the real-world for end-users
~> Transfer of model explanations to explain real-world is generally not permitted

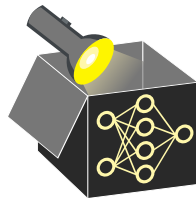


REMARKS: MODEL OR REAL-WORLD

- Most CEs provide explanations of model predictions, but CEs might appear to explain the real-world for end-users
 - ↪ Transfer of model explanations to explain real-world is generally not permitted
- Consider a CE that proposes to increase the feature age by 5 to obtain the loan
 - ↪ a loan applicant takes this information and applies 5 years later for the loan

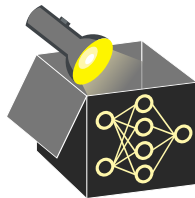


REMARKS: MODEL OR REAL-WORLD



- Most CEs provide explanations of model predictions, but CEs might appear to explain the real-world for end-users
 - ↪ Transfer of model explanations to explain real-world is generally not permitted
- Consider a CE that proposes to increase the feature age by 5 to obtain the loan
 - ↪ a loan applicant takes this information and applies 5 years later for the loan
- However, by then, many other feature values might have changed
 - ↪ not only age, also other causally dependent features e.g. job status might have changed
 - ↪ [Karimi et al. \(2020\)](#) avoid this by considering causal dependencies between features

REMARKS: MODEL OR REAL-WORLD



- Most CEs provide explanations of model predictions, but CEs might appear to explain the real-world for end-users
 - ↪ Transfer of model explanations to explain real-world is generally not permitted
- Consider a CE that proposes to increase the feature age by 5 to obtain the loan
 - ↪ a loan applicant takes this information and applies 5 years later for the loan
- However, by then, many other feature values might have changed
 - ↪ not only age, also other causally dependent features e.g. job status might have changed
 - ↪ [Karimi et al. \(2020\)](#) avoid this by considering causal dependencies between features
- Also, the bank's algorithm might change and previous CEs are not applicable anymore