

Interpretable Machine Learning

Conditional Feature Importance (CFI)

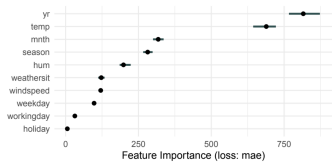
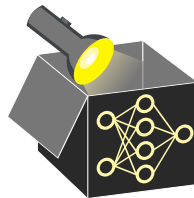


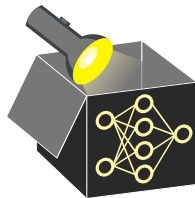
Figure: Bike Sharing Dataset

Learning goals

- Extrapolation and Conditional Sampling
- Conditional Feature Importance (CFI)
- Interpretation of CFI and difference to PFI

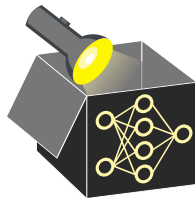
CONDITIONAL FEATURE IMPORTANCE IDEA

- **Permutation Feature Importance Idea:** Replace the feat. of interest x_j with an indep. sample from the marginal dist. $\mathbb{P}(x_j)$, e.g. by randomly perm. obs. in x_j



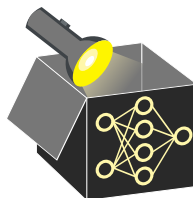
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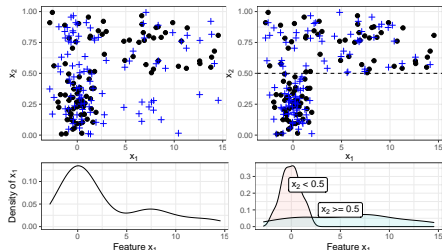
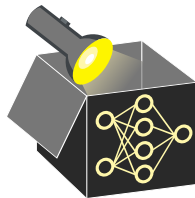
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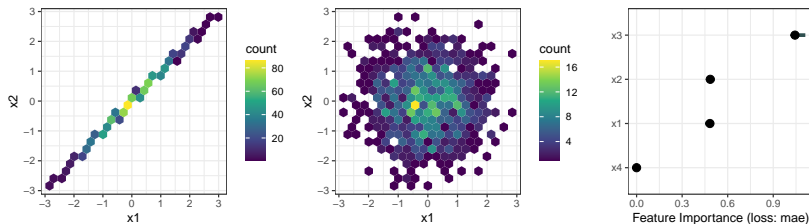
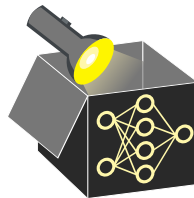
Example: Conditional permutation scheme ▶ Molnar et. al (2020)



- $X_2 \sim U(0, 1)$ and $X_1 \sim N(0, 1)$ if $X_2 < 0.5$, else $X_1 \sim N(4, 4)$ (black dots)
- **Left:** For $X_2 < 0.5$, permuting X_1 (crosses) preserves marginal (but not joint) distribution
~> Bottom: Marginal density of X_1
- **Right:** Permuting X_1 within subgroups $X_2 < 0.5$ & $X_2 \geq 0.5$ reduces extrapolation
~> Bottom: Density of X_1 conditional on groups

RECALL: EXTRAPOLATION IN PFI

Example: Let $y = x_3 + \epsilon_y$ with $\epsilon_y \sim N(0, 0.1)$ where $x_1 := \epsilon_1$, $x_2 := x_1 + \epsilon_2$ are highly correlated ($\epsilon_1 \sim N(0, 1)$, $\epsilon_2 \sim N(0, 0.01)$) and $x_3 := \epsilon_3$, $x_4 := \epsilon_4$, with $\epsilon_3, \epsilon_4 \sim N(0, 1)$. All noise terms are independent. Fitting a LM yields $\hat{f}(\mathbf{x}) \approx 0.3x_1 - 0.3x_2 + x_3$.



Hexbin plot of x_1, x_2 before permuting x_1 (left), after permuting x_1 (center), and PFI scores (right) $\Rightarrow x_1$ and x_2 should be irrelevant for the prediction $\hat{f}(\mathbf{x})$ for

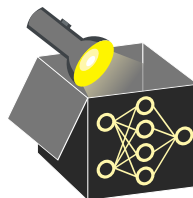
$\{\mathbf{x} : \mathbb{P}(\mathbf{x}) > 0\}$ as $0.3x_1 - 0.3x_2 \approx 0$

\Rightarrow PFI evaluates model on unrealistic obs. outside $\mathbb{P}(\mathbf{x}) \rightsquigarrow x_1$ and x_2 are considered relevant

CONDITIONAL FEATURE IMPORTANCE

► Strobl et al. (2008)

► Hooker et al. (2021)



Conditional feature importance (CFI) for features x_S using test data \mathcal{D} :

- Measure the error **with unperturbed features**.
- Measure the error **with perturbed feature values** $\tilde{x}^{S|-S}$, where $\tilde{x}_S^{S|-S} \sim \mathbb{P}(x_S|x_{-S})$
- Repeat permuting the feature (e.g., m times) and average the difference of both errors:

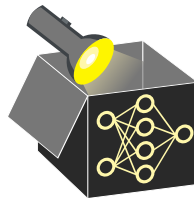
$$\widehat{CFI}_S = \frac{1}{m} \sum_{k=1}^m \mathcal{R}_{\text{emp}}(\hat{f}, \tilde{\mathcal{D}}_{(k)}^{S|-S}) - \mathcal{R}_{\text{emp}}(\hat{f}, \mathcal{D})$$

Here, $\tilde{\mathcal{D}}^{S|-S}$ denotes the dataset where features x_S were sampled conditional on the remaining features x_{-S} .

IMPLICATIONS OF CFI

► König et al. (2020)

Interpretation: Due to the conditional sampling w.r.t. all other features, CFI quantifies a feature's unique contribution to the model performance.



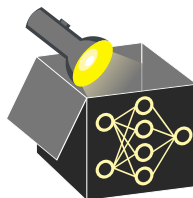
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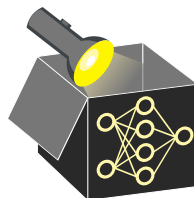
- If feature x_S does not contribute unique information about y , i.e., $x_S \perp\!\!\!\perp y | x_{-S} \Rightarrow \text{CFI} = 0$
- Why? Under the conditional independence $\mathbb{P}(\tilde{x}^{S|-S}, y) = \mathbb{P}(x, y)$
 \rightsquigarrow no prediction-relevant information is destroyed by permutation of x_S conditional on x_{-S}



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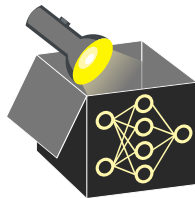
Entanglement with model:

- If the model does not use a feature $\Rightarrow \text{CFI} = 0$
- Why? Then the prediction is not affected by any perturbation of the feature
 \rightsquigarrow model performance does not change after conditional permutation

IMPLICATIONS OF CFI

Can we gain insight into whether ...

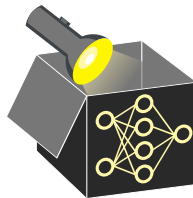
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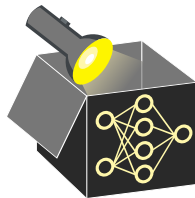
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- ❷ the variable x_j contains prediction-relevant information?
 - If $x_j \not\perp\!\!\!\perp y$ but $x_j \perp\!\!\!\perp y|x_{-j}$ (e.g., x_j and x_{-j} share information) $\Rightarrow CFI_j = 0$
 - x_j is not exploited by model (regardless of whether it is useful for y or not)
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 - x_j is not exploited by model (regardless of whether it is useful for y or not) $\Rightarrow CFI_j = 0$
- ❸ Does the model require access to x_j to achieve its prediction performance?
 - $CFI_j \neq 0 \Rightarrow x_j$ contributes unique information (meaning $x_j \not\perp\!\!\!\perp y|x_{-j}$)
 - Only uncovers the relationships that were exploited by the model



COMPARISON: PFI AND CFI

Example: Let $y = x_3 + \epsilon_y$ with $\epsilon_y \sim N(0, 0.1)$ where $x_1 := \epsilon_1$, $x_2 := x_1 + \epsilon_2$ are highly correlated ($\epsilon_1 \sim N(0, 1)$, $\epsilon_2 \sim N(0, 0.01)$) and $x_3 := \epsilon_3$, $x_4 := \epsilon_4$, with $\epsilon_3, \epsilon_4 \sim N(0, 1)$. All noise terms are independent. Fitting a LM yields $\hat{f}(\mathbf{x}) \approx 0.3x_1 - 0.3x_2 + x_3$.

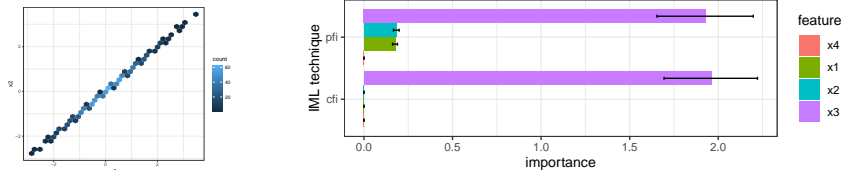
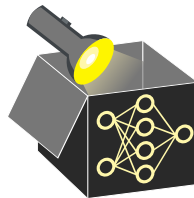


Figure: Density plot for x_1, x_2 before permuting x_1 (left). PFI and CFI (right).

$\Rightarrow x_1$ and x_2 are irrelevant for the prediction $\hat{f}(\mathbf{x})$ for $\{\mathbf{x} : \mathbb{P}(\mathbf{x}) > 0\}$ as

$$0.3x_1 - 0.3x_2 \approx 0$$

\Rightarrow PFI evaluates model on unrealistic obs. outside $\mathbb{P}(\mathbf{x}) \rightsquigarrow x_1, x_2$ are considered relevant (PFI > 0)

\Rightarrow Since x_1 can be reconstructed from x_2 and vice versa, CFI considers x_1 and x_2 to be irrelevant