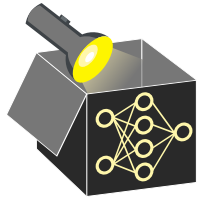
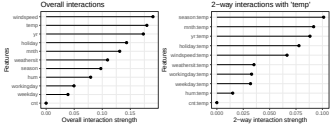
Interpretable Machine Learning



Friedman's H-Statistic

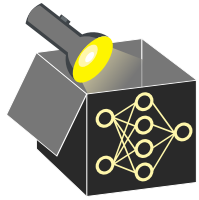


Learning goals

- Understand Friedman's H-statistic
- Measure 2-way interactions between pairs of features
- Measure a feature's overall interaction strength

IDEA

▸ Friedman and Popescu (2008)

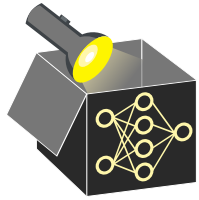


2-way interaction: If two features j and k do not interact, their mean-centered PD function is

$$\hat{f}_{jk,PD}(x_j, x_k) = \hat{f}_{j,PD}(x_j) + \hat{f}_{k,PD}(x_k)$$

- $\hat{f}_{jk,PD}(x_j, x_k)$: joint 2-dim PD function of feature j and k
- $\hat{f}_{j,PD}(x_j)$ and $\hat{f}_{k,PD}(x_k)$: 1-dim PD functions of single features j and k

IDEA

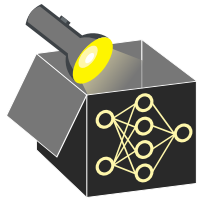


Overall interaction: If feature j does not interact with any other feature (denoted by index $-j$), the mean-centered prediction function can be decomposed by

$$\hat{f}(\mathbf{x}) = \hat{f}_{j,PD}(x_j) + \hat{f}_{-j,PD}(\mathbf{x}_{-j})$$

- $\hat{f}(\mathbf{x})$: mean-centered prediction function
- $\hat{f}_{j,PD}(x_j)$: 1-dim PD function of feature j
- $\hat{f}_{-j,PD}(\mathbf{x}_{-j})$: $(p-1)$-dim PD function of all p features except feature j

2-WAY INTERACTION STRENGTH

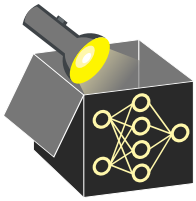


H-statistic measures interaction strength between feature j and k by

$$H_{jk}^2 = \frac{\sum_{i=1}^{n} \left[\hat{f}_{jk,PD}(x_j^{(i)}, x_k^{(i)}) - \hat{f}_{j,PD}(x_j^{(i)}) - \hat{f}_{k,PD}(x_k^{(i)}) \right]^2}{\sum_{i=1}^{n} \left[\hat{f}_{jk,PD}(x_j^{(i)}, x_k^{(i)}) \right]^2}$$

Note: The numerator is 0 if the two features x_j and x_k do not interact, i.e.,
$\hat{f}_{jk,PD}(x_j, x_k) - \hat{f}_{j,PD}(x_j) - \hat{f}_{k,PD}(x_k) = 0$.
$\Rightarrow$ The smaller the values of H_{jk}^2, the weaker the interaction between x_j and x_k.

OVERALL INTERACTION STRENGTH



Similarly, it is possible to measure whether a feature j interacts with any other feature (Overall interaction strength):

$$H_j^2 = \frac{\sum_{i=1}^{n} \left[\hat{f}(x^{(i)}) - \hat{f}_{j,PD}(x_j^{(i)}) - \hat{f}_{-j,PD}(x_{-j}^{(i)}) \right]^2}{\sum_{i=1}^{n} \left[\hat{f}(x^{(i)}) \right]^2}$$

Example: Inspect interactions of a random forest for the bike data