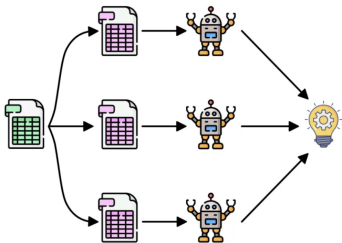
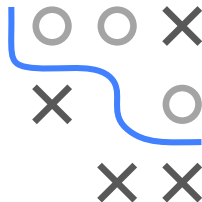


# Introduction to Machine Learning

## Random Forest

## Bagging Ensembles

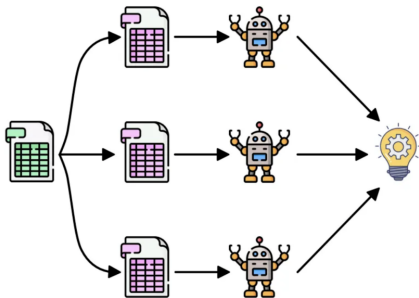
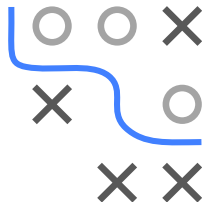


### Learning goals

- Understand idea of bagging
- Be able to explain the connection between bagging and bootstrap
- Understand why bagging improves predictive performance

# BAGGING

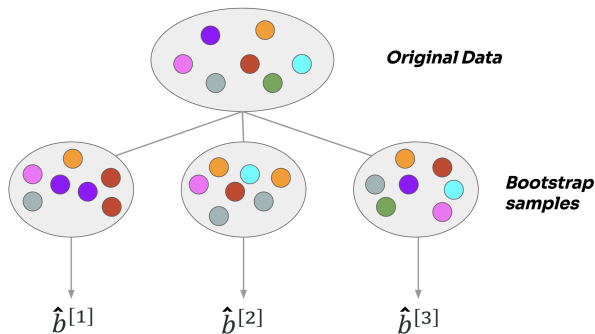
- Bagging is short for **B**ootstrap **A**ggregation
- **Ensemble method**, combines models into large “meta-model”; ensembles usually better than single **base learner**
- Homogeneous ensembles always use same BL class (e.g. CART), heterogeneous ensembles can use different classes
- Bagging is homogeneous



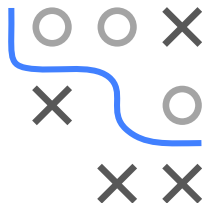
# TRAINING BAGGED ENSEMBLES

Train BL on  $M$  **bootstrap** samples of training data  $\mathcal{D}$ :

- Draw  $n$  observations from  $\mathcal{D}$  with replacement
- Fit BL on each bootstrapped data  $\mathcal{D}^{[m]}$  to obtain  $\hat{b}^{[m]}$

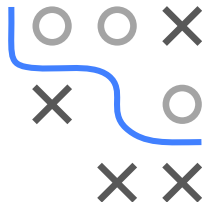
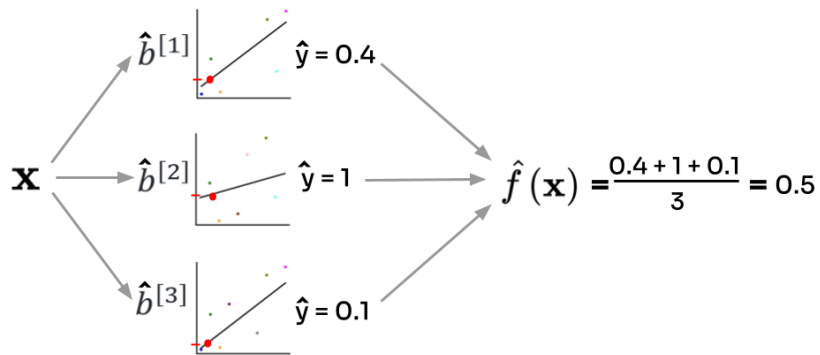


- Data sampled in one iter called “in-bag” (IB)
- Data not sampled called “out-of-bag” (OOB)



# PREDICTING WITH A BAGGED ENSEMBLE

**Average** predictions of  $M$  fitted models for ensemble:  
(here: regression)



# BAGGING PSEUDO CODE

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## Bagging algorithm: Training

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- 1: **Input:** Dataset  $\mathcal{D}$ , type of BLs, number of bootstraps  $M$
  - 2: **for**  $m = 1 \rightarrow M$  **do**
  - 3:     Draw a bootstrap sample  $\mathcal{D}^{[m]}$  from  $\mathcal{D}$
  - 4:     Train BL on  $\mathcal{D}^{[m]}$  to obtain model  $\hat{b}^{[m]}$
  - 5: **end for**
- 

## Bagging algorithm: Prediction

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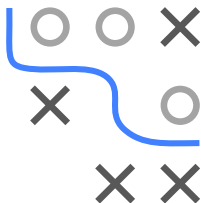
- 1: **Input:** Obs.  $\mathbf{x}$ , trained BLs  $\hat{b}^{[m]}$  (as scores  $\hat{f}^{[m]}$ , hard labels  $\hat{h}^{[m]}$  or probs  $\hat{\pi}^{[m]}$ )
- 2: Aggregate/Average predictions

$$\hat{f}(\mathbf{x}) = \frac{1}{M} \sum_{m=1}^M \left( \hat{f}^{[m]}(\mathbf{x}) \right) \quad (\text{regression / decision score, use } \hat{f}_k \text{ in multi-class})$$

$$\hat{h}(\mathbf{x}) = \arg \max_{k \in \mathcal{Y}} \sum_{m=1}^M \mathbb{I} \left( \hat{h}^{[m]}(\mathbf{x}) = k \right) \quad (\text{majority voting})$$

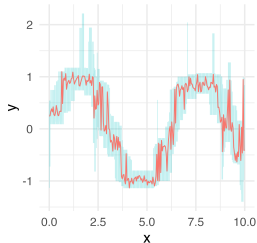
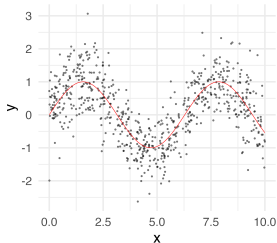
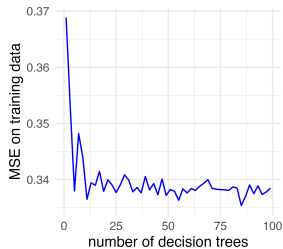
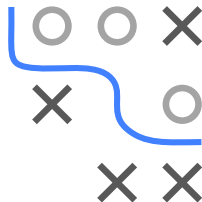
$$\hat{\pi}_k(\mathbf{x}) = \begin{cases} \frac{1}{M} \sum_{m=1}^M \hat{\pi}_k^{[m]}(\mathbf{x}) & (\text{probabilities through averaging}) \\ \frac{1}{M} \sum_{m=1}^M \mathbb{I} \left( \hat{h}^{[m]}(\mathbf{x}) = k \right) & (\text{probabilities through class frequencies}) \end{cases}$$

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# WHY/WHEN DOES BAGGING HELP?

- Bagging reduces the variability of predictions by averaging the outcomes from multiple BL models
- It is particularly effective when the errors of a BL are mainly due to (random) variability rather than systematic issues



- Increasing **nr. of BLs** improves performance, up to a point, optimal ensemble size depends on inducer and data distribution

# MINI BENCHMARK

Bagged ensembles with 100 BLs each on spam:

Bagging seems especially helpful for less stable learners like CART

