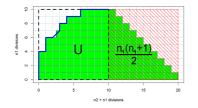
Introduction to Machine Learning

Evaluation AUC & Mann-Whitney-U Test



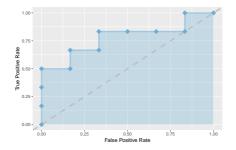


Learning goals

- Understand the rank-based nature of AUC
- See the connection between AUC and Mann-Whitney-U statistic

AUC AS A RANK-BASED METRIC

- The AUC metric is intimately related to the Mann-Whitney-U test, also known as Wilcoxon rank-sum test.
- This connection is best understood viewing the AUC from a slightly different angle: it is, in effect, a rank-based metric.
- Recall that, constructing the ROC curve, we count TP and FP.

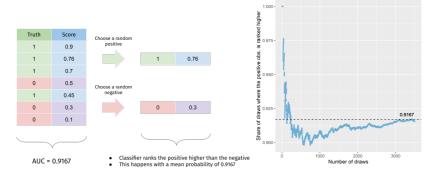


• The AUC abstracts from the actual classification scores and considers only their rank.



AUC AS A RANK-BASED METRIC /2

- We can interpret the AUC as the probability of our classifier ranking a random positive observation higher than a random negative one.
- A perfect classifier will rank all positive above all negative observations, achieving AUC = 1.





MANN-WHITNEY-U TEST

- The Mann-Whitney-U test is a **non-parametric hypothesis test** on the difference in location between two samples X_1 , X_2 of sizes n_1 and n_2 , respectively.
- Under the null, X_1 and X_2 follow the same (unknown) distribution \mathbb{P} , and for any pair of observations $x_{1,1} \in X_1$, $x_{2,1} \in X_2$ drawn at random from \mathbb{P} , the following statement holds: $\mathbb{P}(x_{1,1} \in X_1) > \mathbb{P}(x_{2,1} \in X_2) = \mathbb{P}(x_{1,1} \in X_1) < \mathbb{P}(x_{2,1} \in X_2) = 0.5$.
- The test statistic estimates the probability of a random sample from X_1 ranking higher than one from X_2 (R_1 denoting the sum of ranks of the $x_{1,i}$):

$$U = \frac{1}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{i=1}^{n_2} \mathbb{I}[x_{1,i} > x_{2,j}] = R_1 - \frac{n_1(n_1 + 1)}{2}$$

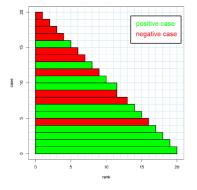
• For large samples, *U* is approximately normally distributed.

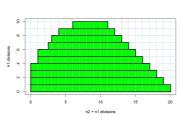


AUC & MANN-WHITNEY-U TEST

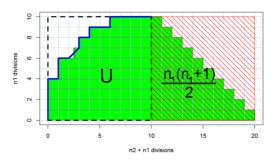
- We can directly interpret the AUC in the light of the U statistic.
- In order to see this, plot the ranks of all the scores as a stack of horizontal bars, and color them by label.
- Next, keep only the green ones, and slide them horizontally to get a nice even stairstep on the right edge:







AUC & MANN-WHITNEY-U TEST / 2



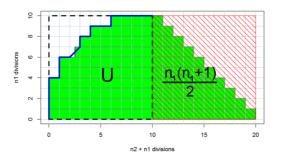


• Denoting the respective numbers of cases as n_+ and n_- , we have:

$$U=R_{+}-rac{n_{+}(n_{+}+1)}{2}.$$

• The area of the green bars on the right is equal to $\frac{n_+(n_++1)}{2}$.

AUC & MANN-WHITNEY-U TEST / 3





- *U*: area of the green bars on the left.
- $n_+ \cdot n_-$: area of the dashed rectangle.
- \Rightarrow AUC is *U* normalized to the unit square:

$$\mathsf{AUC} = \frac{\mathit{U}}{\mathit{n}_+ \cdot \mathit{n}_-}$$