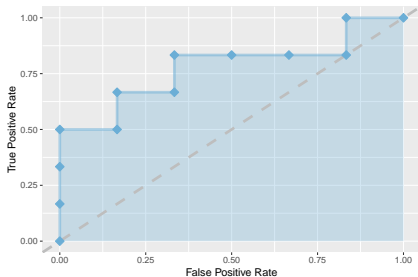
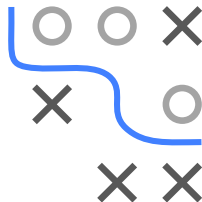


AUC AS A RANK-BASED METRIC

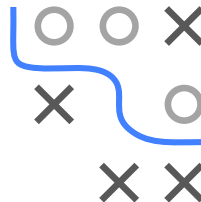
- The AUC metric is intimately related to the **Mann-Whitney-U test**, also known as **Wilcoxon rank-sum test**.
- This connection is best understood viewing the AUC from a slightly different angle: it is, in effect, a **rank-based** metric.
- Recall that, constructing the ROC curve, we count TP and FP.



- The AUC abstracts from the actual classification scores and considers only their rank.

AUC AS A RANK-BASED METRIC / 2

- We can interpret the AUC as the probability of our classifier ranking a random positive observation higher than a random negative one.
- A perfect classifier will rank all positive above all negative observations, achieving $AUC = 1$.



Truth	Score
1	0.9
1	0.76
1	0.7
0	0.5
1	0.45
0	0.3
0	0.1

Choose a random positive



1	0.76
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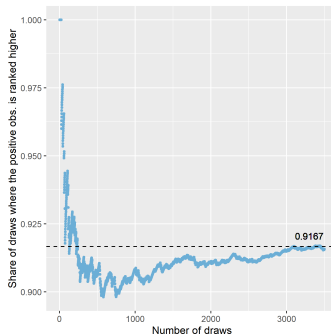
Choose a random negative



0	0.3
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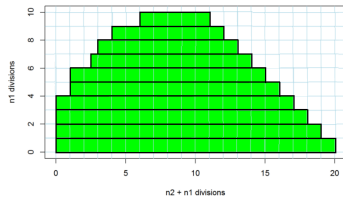
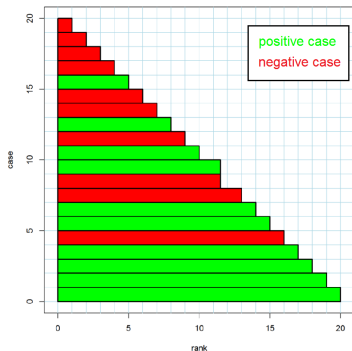
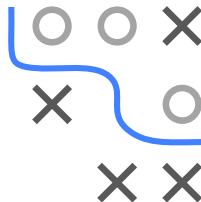
AUC = 0.9167

- Classifier ranks the positive higher than the negative
- This happens with a mean probability of 0.9167

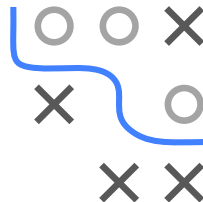
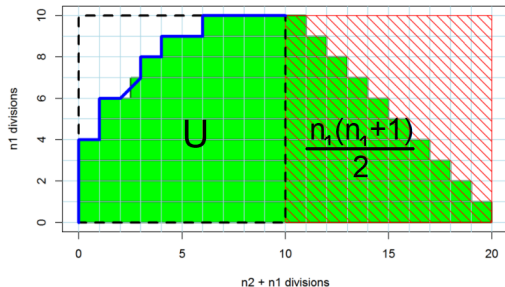


AUC & MANN-WHITNEY-U TEST

- We can directly interpret the AUC in the light of the U statistic.
- In order to see this, plot the ranks of all the scores as a stack of horizontal bars, and color them by label.
- Next, keep only the green ones, and slide them horizontally to get a nice even stairstep on the right edge:



AUC & MANN-WHITNEY-U TEST / 2

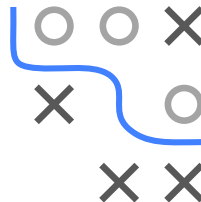
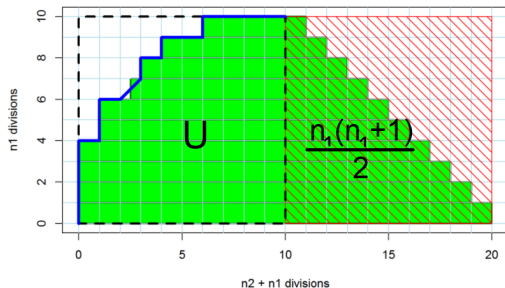


- Denoting the respective numbers of cases as n_+ and n_- , we have:

$$U = R_+ - \frac{n_+(n_+ + 1)}{2}.$$

- The area of the green bars on the right is equal to $\frac{n_+(n_+ + 1)}{2}$.

AUC & MANN-WHITNEY-U TEST / 3



- U : area of the green bars on the left.
- $n_+ \cdot n_-$: area of the dashed rectangle.

⇒ AUC is U normalized to the unit square:

$$\text{AUC} = \frac{U}{n_+ \cdot n_-}.$$