COST CURVES

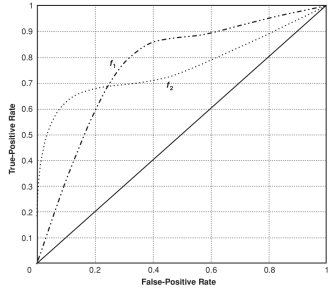
- Directly plot the misclassif costs / error (in terms of prior probs)
- Might be easier to interpret than ROC, especially in case of different misclassif costs or priors

Example:

- f_1 and f_2 with intersecting ROC curves
- f_2 dominates first, then f_1

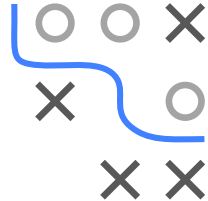
BUT: Unclear for which thresholds, costs or class distribs f_2 better than f_1

ROC curves for f_1 and f_2



Nathalie Japkowicz (2004): Evaluating Learning Algorithms : A Classification Perspective. (p. 125)

COST CURVES



Simplifying assumption: equal misclassif costs, i.e., $cost_{FN} = cost_{FP}$
$\Rightarrow$ Expected misclassif cost reduces to misclassif error rate
With law of total prob, we write error rate as function of π_+:

$$\begin{aligned}\rho_{MCE}(\pi_+) &= (1-\pi_+)\cdot \mathbb{P}(\hat{y}=1|y=0) + \pi_+ \cdot \mathbb{P}(\hat{y}=0|y=1)\\ &= (1-\pi_+)\cdot FPR + \pi_+ \cdot FNR\\ &= (FNR - FPR)\cdot \pi_+ + FPR\end{aligned}$$

Confusion matrix

		True class	
		$y=1$	$y=0$
Pred. class	$\hat{y}=1$	TP	FP
	$\hat{y}=0$	FN	TN

Cost matrix

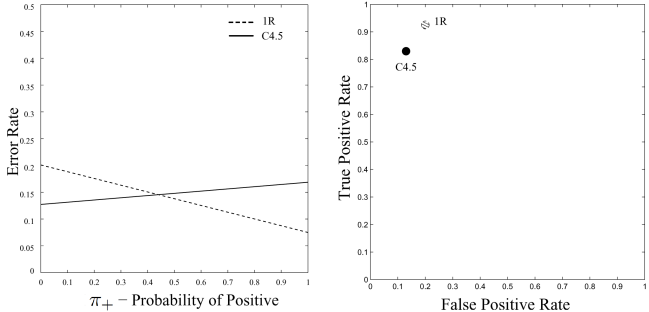
		True class	
		$y=1$	$y=0$
Pred. class	$\hat{y}=1$	0	$cost_{FP}$
	$\hat{y}=0$	$cost_{FN}$	0

COST CURVES

- Cost line of a classifier with slope $(FNR - FPR)$ and intercept FPR:

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

- Cost curves are point–line duals of ROC curves, i.e., a single classifier is represented by a point in the ROC space and by a line in cost space



Chris Drummond and Robert C. Holte (2006): Cost curves: An improved method for visualizing classifier performance. Machine Learning, 65, 95-130 (URL).

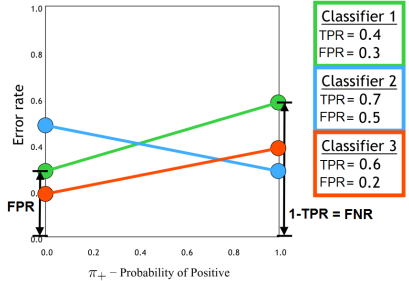
COST LINES

Cost line of a classifier with slope $(FNR - FPR)$ and intercept FPR:

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

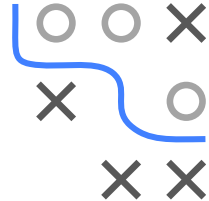
- Hard classifiers are points (TPR, FPR) in ROC space
- The cost line of a classifier connects (π_+, ρ_{MCE})-points at $(0, FPR)$ and $(1, 1 - TPR)$
- Classifier 3 always dominates classifier 1
- Classifier 3 is better than classifier 2 when $\pi_+ < 0.7$

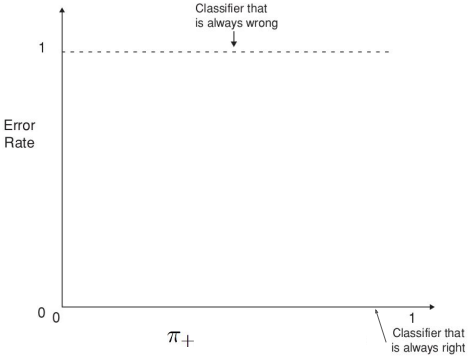
Cost lines plot different values of π_+ vs. $\rho_{MCE}(\pi_+)$



COST LINES - EXAMPLE

- Horizontal dashed line: worst classifier (100% error rate for all π_+) $\Rightarrow FNR = FPR = 1$
- x-axis: perfect classifier (0% error rate for all π_+) $\Rightarrow FNR = FPR = 0$





$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

Confusion matrix

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class	$\hat{y} = 0$	FN	TN

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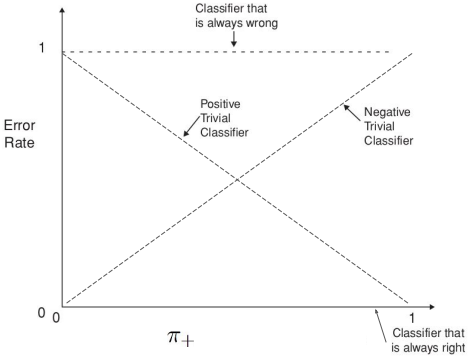
- Dashed diagonal lines: trivial classifiers, i.e., ascending diagonal always predicts negative instances ($\Rightarrow FNR = 1$ and $FPR = 0$) and vice versa



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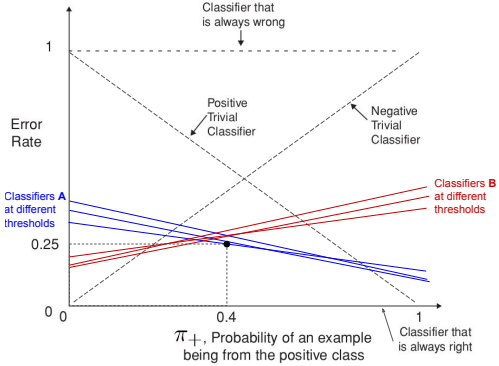
- Descending/ascending bold lines: two families of classifiers A and B (represented by points in their respective ROC curves)



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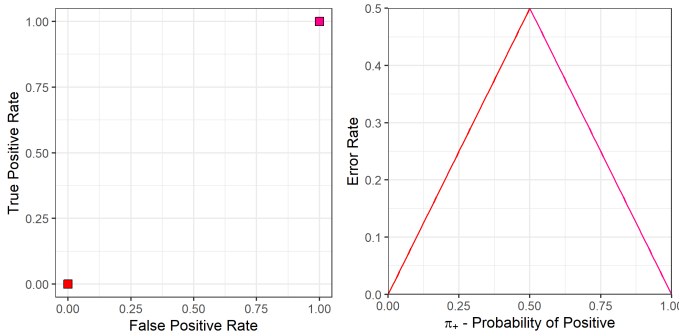
VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.



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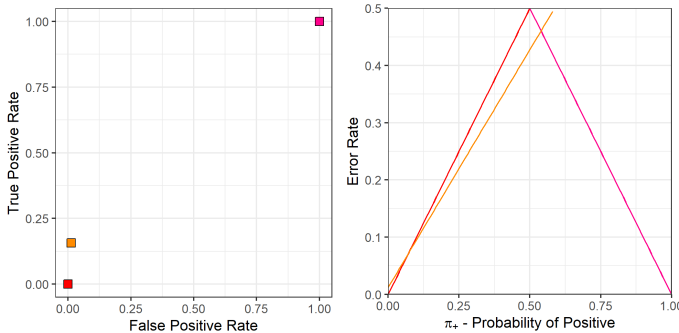

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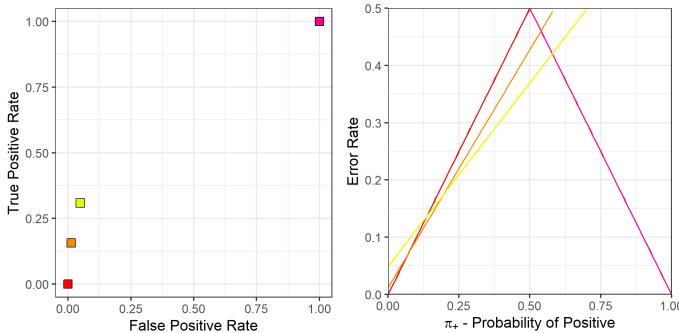

VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.
- **Cost curve** (right, black) is lower envelope of **cost lines**
  $\hat{=}$ pointwise minimum of error rate (as function of π_+)