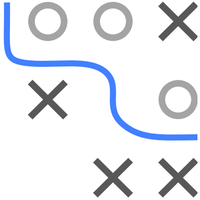
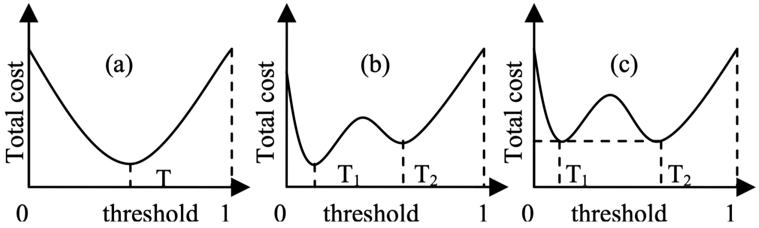
EMPIRICAL THRESHOLDING: BINARY CASE



- Theoretical threshold from MECP not always best, due to e.g. wrong model class, finite data, etc.
- Simply measure costs on data with different thresholds
- Then pick best threshold (Fig.1 in ▸ Sheng et al. 2006):



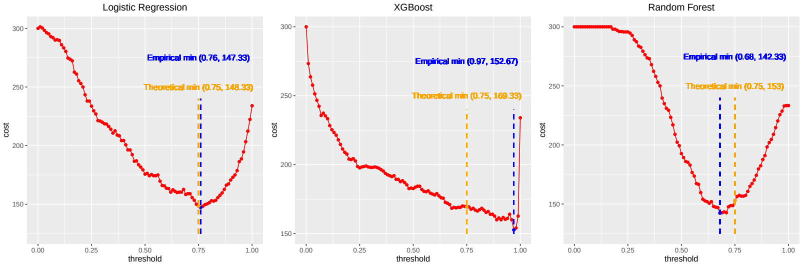
- What if two equal local minima? We prefer the one with wider span
- Do this on validation data / over cross-val to avoid overfitting!

EMPIRICAL THRESHOLDING: BINARY CASE

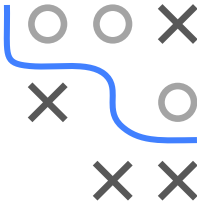
- Example: German Credit task

		True class	
		$y = \text{good}$	$y = \text{bad}$
Pred.	$\hat{y}$ = good	0	3
class	$\hat{y}$ = bad	1	0

- Theoretical: $C(good, bad)/(C(bad, good) + C(good, bad)) = 3/4 = c^*$
- Empirical version with 3-CV: For XGBoost, empirical minimum deviates substantially from theoretical version



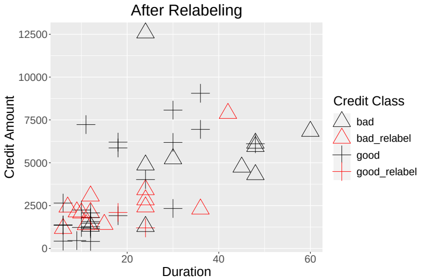
EMPIRICAL THRESHOLDING: MULTICLASS



- In the standard setting, we predict class $h(\mathbf{x}) = \arg\max_k \pi_k(\mathbf{x})$.
- Let's use g thresholds c_k now
- Re-scale scores $\mathbf{s} = (\frac{\pi(\mathbf{x})_1}{c_1}, \dots, \frac{\pi(\mathbf{x})_g}{c_g})^\top$,
- Predict class $\arg\max_k \pi_k(\mathbf{x})$.
- Compute empirical costs over cross-validation
- Optimize over g (actually: $g - 1$) dimensional threshold vector $(c_1, \dots, c_g)^T$ to produce minimal costs

METACOST: OVERVIEW

- Model-agnostic wrapper technique
- General idea:
  1. Relabel train obs with their low expected cost classes
  2. Apply classifier to relabeled data
- Example German Credit task:



- Relabeled instances colored red
- Relabeling from good to bad more common because of costs

METACOST: ALGORITHM

Input: $\mathcal{D} = \{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^{n}$ training data, B number of bagging iterations, $\pi(\mathbf{x})$ probabilistic classifier, $\mathbf{C}$ cost matrix, empty dataset $\tilde{D} = \emptyset$

\# Bagging: Classifier is trained on different bootstrap samples.

for $b = 1, \dots, B$ **do**

  $\mathcal{D}_b \leftarrow$ Bootstrap version of $\mathcal{D}$

  $\pi_b \leftarrow$ train classifier on $\mathcal{D}_b$

end for

\# Relabeling: Find classifiers for which $\mathbf{x}^{(i)}$ is OOB and compute π_b by averaging over predictions. Determine new label $\tilde{y}^{(i)}$ w.r.t. to the cost minimal class.

for $i = 1, \dots, n$ **do**

  $\tilde{M} \leftarrow \bigcup_{m:\mathbf{x}^{(i)} \notin \mathcal{D}_m} \{m\}$

end for

for $j = 1, \dots, g$ **do**

  $\pi_j(\mathbf{x}^{(i)}) \leftarrow \frac{1}{|\tilde{M}|} \sum_{m \in \tilde{M}} \pi_j(\mathbf{x}^{(i)} \mid f_m)$ for each i

end for

for $i = 1, \dots, n$ **do**

  $\tilde{y}^{(i)} \leftarrow \arg\min_k \sum_{j=1}^{g} \pi_j(\mathbf{x}^{(i)}) C(k, j)$

  $\tilde{D} \leftarrow \tilde{D} \cup \{(\mathbf{x}^{(i)}, \tilde{y}^{(i)})\}$

end for

\# Cost Sensitivity: Train on relabeled data.

$f_{meta} \leftarrow$ train f on $\tilde{D}$