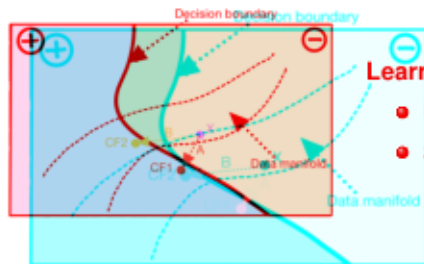


Interpretable Machine Learning

Counterfactual Explanations



Learning goals

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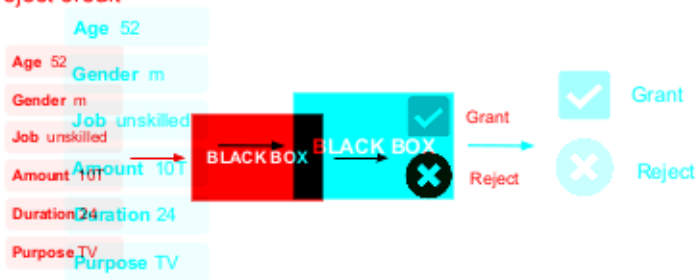
- Understand the motivation behind CEs
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- See the mathematical foundation of CEs
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EXAMPLE: CREDIT RISK APPLICATION

- x : customer and credit information

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- y : grant or reject credit



Questions:

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- Why was the credit rejected?

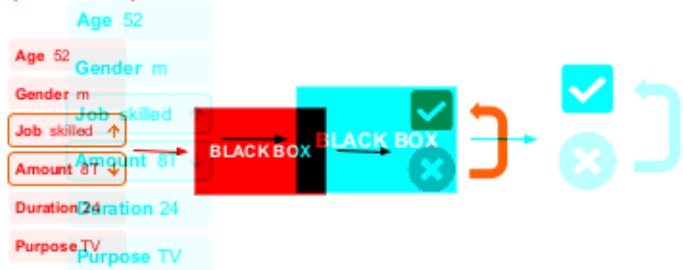
- Is it a fair decision?

- How should x be changed so that the credit is accepted?

EXAMPLE: CREDIT RISK APPLICATION



Counterfactual Explanations provide answers in the form of "What-If"-scenarios.



"If the person was more skilled and the credit amount had been reduced to \$8,000, the credit would have been granted."

COUNTERFACTUAL EXPLANATIONS: MAIN IDEA

- Counterfactual explanations == counterfactuals == CEs



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AIMS & ROLES

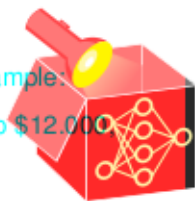
CEs can serve various purposes; the user can decide what to learn from them. For example:

"If the person had been **one year older** and the **credit amount had been increased** to \$12,000,

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There is a bug, an increase in amount should not increase approval rates.

- **Detect model biases:**

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PHILOSOPHICAL BASIS

Counterfactuals have a long-standing tradition in analytic philosophy

~ According to Lewis (1975), a **counterfactual conditional** is a statement of the form:

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- A world is similar to another if laws are maximally preserved between the worlds and only a few facts are changed



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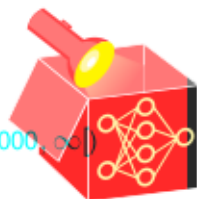


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 - ~> e.g., decreasing loan amount by \$20.000 and being one year older is recommended by the explainer, although only loan amount might be causally relevant
- CEs are often contrastive, i.e., they explain a decision by referring to an alternative outcome
 - ~> e.g., if the loan applicant was 30 instead of 60 years old, the approved loan would have been over \$100.000 instead of \$40.000

MATHEMATICAL PERSPECTIVE

Terminology:

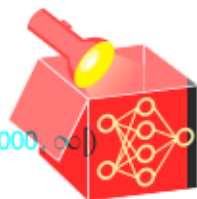
- x : original/factual datapoint whose prediction we want to explain
- $y' \subset \mathbb{R}$: desired prediction ($y' \neq 1000$ or $y' \Rightarrow$ "grant credit") or interval ($y' = [1000, \infty)$)
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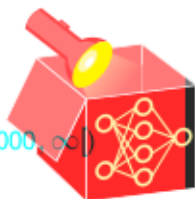
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$$\arg \min_{\mathbf{x}'} \lambda_1 o_p(\hat{f}(\mathbf{x}'), y') + \lambda_2 o_f(\mathbf{x}', \mathbf{x})$$

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- Regression: o_p could be the L_1 -distance $o_p(\hat{f}(\mathbf{x}'), y') = |\hat{f}(\mathbf{x}') - y'|$

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$$o_f(\mathbf{x}', \mathbf{x}) = d_G(\mathbf{x}', \mathbf{x}) = \frac{1}{p} \sum_{j=1}^p \delta_G(x'_j, x_j) \in [0, 1]$$

The value of δ_G depends on the feature type (numerical or categorical):
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$$\delta_G(x'_j, x_j) = \begin{cases} \frac{1}{\hat{R}_j} |x'_j - x_j| & \text{if } x_j \text{ is numerical} \\ \mathcal{I}_{\{x'_j \neq x_j\}} & \text{if } x_j \text{ is categorical} \end{cases}$$

with \hat{R}_j as the value range of feature j in the training dataset (to ensure that $\delta_G(x'_j, x_j) \in [0, 1]$)
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FURTHER OBJECTIVES

Additional constraints can improve the explanation quality of the corresponding CEs
~ popular constraints include sparsity and plausibility

Sparsity:

- End users often prefer short over long explanations
- ~ counterfactuals should be **sparse**



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- Independently from o_7 , sparsity in the changes can be additionally considered by another objective that counts the number of changed features via the L_0 -norm:

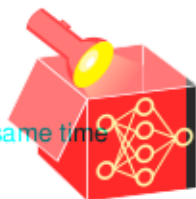
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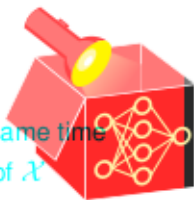
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Example from Verma et al. (2020)

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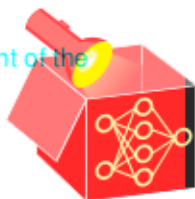
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 - Path A for CF1 is shorter
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 - Path B for CF2 is longer but adheres to data manifold
- Path A for CF1 is shorter
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FURTHER OBJECTIVES

To ensure plausibility, o_4 could, e.g., be the Gower distance of \mathbf{x}' to its nearest data point of the training dataset which we denote $\mathbf{x}^{[1]}$:

$$o_4(\mathbf{x}', \mathbf{X}) = d_G(\mathbf{x}', \mathbf{x}^{[1]}) = d_G\left(\frac{1}{p} \sum_{j=1}^p \delta_G(x'_j, x_j^{[1]})\right)$$



We can extend the previous optimization problem by adding o_s (for sparsity) and o_4 (for plausibility):

$$\arg \min_{\mathbf{x}'} \lambda_1 o_p(\hat{f}(\mathbf{x}'), y') + \lambda_2 o_f(\mathbf{x}', \mathbf{x}) + \lambda_3 o_s(\mathbf{x}', \mathbf{x}) + \lambda_4 o_4(\mathbf{x}', \mathbf{X})$$

REMARKS: THE RASHOMON EFFECT

Issue (Rashomon effect):

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Note:

- As the model is generally non-linear, inconsistent and diverse CEs can arise
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- How to deal with the Rashomon effect is considered an open problem in IML

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