BINARY INSTANCE-SPECIFIC COST LEARNING

- Assumes instance-specific costs for every observation: $\mathcal{D}^{(n)} = \{(\mathbf{x}^{(i)}, \mathbf{c}^{(i)})\}_{i=1}^{n}$, where $(\mathbf{x}^{(i)}, \mathbf{c}^{(i)}) \in \mathbb{R}^p \times \mathbb{R}^2$.
- Define "true class" as cost minimal class
- Define observation weights: $|\mathbf{c}^{(i)}[1] - \mathbf{c}^{(i)}[0]|$

	$\mathbf{c}^{(i)}[0]$	$\mathbf{c}^{(i)}[1]$	$y^{(i)}$	$w^{(i)}$
$\mathbf{x}^{(1)}$	1	1	0	0
$\mathbf{x}^{(2)}$	1	2	0	1
$\mathbf{x}^{(3)}$	7	3	1	4

- Now solve weighted ERM:

$$\mathcal{R}_{emp}(\theta) = \sum_{i=1}^{n} w^{(i)} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)} \middle| \theta\right)\right)$$

- NB: Instances with equal costs are effectively ignored.

Advanced Machine Learning

Imbalanced Learning: Cost-Sensitive Learning Part 3

Confusion matrix

		True class	
		$y = 1$	$y = -1$
Pred.	$\hat{y} = 1$	TP	FP
class	$\hat{y} = -1$	FN	TN

Cost matrix

		True class	
		$y = 1$	$y = -1$
Pred.	$\hat{y} = 1$	$C(1, 1)$	$C(1, -1)$
class	$\hat{y} = -1$	$C(-1, 1)$	$C(-1, -1)$

Learning goals

- Instance specific costs
- Cost-sensitive OVO

INSTANCE-SPECIFIC COST LEARNING

- Consider instance-specific costs for binary classification: $\mathcal{D}^{(c)} = \left\{\left(\mathbf{x}^{(i)}, \mathbf{c}^{(i)}\right)\right\}_{i=1}^{n}$, with $\left(\mathbf{x}^{(i)}, \mathbf{c}^{(i)}\right) \in \mathbb{R}^p \times \mathbb{R}^2$.

	$\mathbf{c}^{(i)}[0]$	$\mathbf{c}^{(i)}[1]$	$y^{(i)}$	$w^{(i)}$
$\mathbf{x}^{(1)}$	1	1	0	0
$\mathbf{x}^{(2)}$	1	2	0	1
$\mathbf{x}^{(3)}$	7	3	1	4

- Define "true class" as cost minimal class
- Define observation weights: $\left|\mathbf{c}^{(i)}[1] - \mathbf{c}^{(i)}[0]\right|$
- Now solve weighted ERM:

$$\mathcal{R}_{emp}(\theta) = \sum_{i=1}^{n} w^{(i)} L\left(y^{(i)}, f\left(\mathbf{x}^{(i)} \mid \theta\right)\right)$$

- NB: Instances with equal costs are effectively ignored.

MULTICLASS COSTS

- Consider $g > 2$. Vanilla CSL is special case of instance specific, use $\mathbf{c}^{(i)}$ same for all $\mathbf{x}^{(i)}$ of the same class

Pred. class	True class $y = 1$	$y = 2$	$y = 3$
$\hat{y} = 1$	0	1	3
$\hat{y} = 2$	1	0	1
$\hat{y} = 3$	7	1	0

- For two $\mathbf{x}^{(i)}$ with $y = 2$ and $y = 3$:

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$	$y^{(i)}$
$\mathbf{x}^{(1)}$	1	0	1	2
$\mathbf{x}^{(2)}$	3	1	0	3
$\mathbf{x}^{(3)}$	1	0	1	2

- Set $\mathbf{c}^{(i)}[y^{(i)}] = 0$, i.e. zero-cost for correct prediction.

CSOVO [LIN ET AL. 2014]

- Let $\mathcal{D}^{(n)} = \{(\mathbf{x}^{(i)}, \mathbf{c}^{(i)})\}_{i=1}^{n}$, $(\mathbf{x}^{(i)}, \mathbf{c}^{(i)}) \in \mathbb{R}^p \times \mathbb{R}^g$.
- Example:

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$
$\mathbf{x}^{(1)}$	0	2	3
$\mathbf{x}^{(2)}$	1	0	1
$\mathbf{x}^{(3)}$	2	0	3

- Idea: Reduction principle to binary case (weighted fit) by one-versus-one (OVO).
- For class j vs. k:
  - How to deal with the label $y^{(i)}$? $y^{(i)}$ can be neither j nor k.
  - How to deal with the costs $\mathbf{c}^{(i)}[j]$ and $\mathbf{c}^{(i)}[k]$?

MULTICLASS COSTS

- Consider $g > 2$. Vanilla CSL is special case of instance specific, use $\mathbf{c}^{(i)}$ same for all $\mathbf{x}^{(i)}$ of the same class

		True class		
		$y = 1$	$y = 2$	$y = 3$
Pred. class	$\hat{y} = 1$	0	1	3
	$\hat{y} = 2$	1	0	1
	$\hat{y} = 3$	7	1	0

- For two $\mathbf{x}^{(i)}$ with $y = 2$ and $y = 3$:

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$	$y^{(i)}$
$\mathbf{x}^{(1)}$	1	0	1	2
$\mathbf{x}^{(2)}$	3	1	0	3
$\mathbf{x}^{(3)}$	1	0	1	2

- Set $\mathbf{c}^{(i)}[y^{(i)}] = 0$, i.e. zero-cost for correct prediction.

CSOVO

▸ Lin et al. 2014

- Let $\mathcal{D}^{(c)} = \{(\mathbf{x}^{(i)}, \mathbf{c}^{(i)})\}_{i=1}^{n}$, $(\mathbf{x}^{(i)}, \mathbf{c}^{(i)}) \in \mathbb{R}^{p} \times \mathbb{R}^{g}$
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- Idea: Reduction principle to binary case (weighted fit) by one-versus-one (OVO).
- For class j vs. k:
  - How to deal with the label $y^{(i)}$? $y^{(i)}$ can be neither j nor k
  - How to deal with the costs $\mathbf{c}^{(i)}[j]$ and $\mathbf{c}^{(i)}[k]$?

- When training a binary classifier $f^{(j,k)}$ for class j vs. k,
  - Choose cost min class from pair $\arg\min_{l \in \{j,k\}} \mathbf{c}^{(i)}[l]$ as ground truth
  - Sample weight is simply diff between the 2 costs $|\mathbf{c}^{(i)}[j] - \mathbf{c}^{(i)}[k]|$
- Example continued:

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$	$\mathbf{c}^{(i)}[1 \text{ vs } 2]$	$\tilde{y}^{(i)}[1 \text{ vs } 2]$	$w^{(i)}[1 \text{ vs } 2]$
$\mathbf{x}^{(1)}$	0	2	3	0/2	1	2
$\mathbf{x}^{(2)}$	1	0	1	1/0	2	1
$\mathbf{x}^{(3)}$	2	0	3	2/0	2	2

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$	$\mathbf{c}^{(i)}[2 \text{ vs } 3]$	$\tilde{y}^{(i)}[2 \text{ vs } 3]$	$w^{(i)}[2 \text{ vs } 3]$
$\mathbf{x}^{(1)}$	0	2	3	2/3	2	1
$\mathbf{x}^{(2)}$	1	0	1	0/1	2	1
$\mathbf{x}^{(3)}$	2	0	3	0/3	2	3

- Example continued

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$	$\mathbf{c}^{(i)}[1 \text{ vs } 3]$	$\tilde{y}^{(i)}[1 \text{ vs } 3]$	$w^{(i)}[1 \text{ vs } 3]$
$\mathbf{x}^{(1)}$	0	2	3	0/3	1	3
$\mathbf{x}^{(2)}$	1	0	1	-/-	-	0
$\mathbf{x}^{(3)}$	2	0	3	2/3	1	1

- When training binary classifier $f^{(j,k)}$ for class j vs. k,
  - Choose cost-min class from pair $\arg\min_{l \in \{j,k\}} \mathbf{c}^{(i)}[l]$ as ground truth
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$\mathbf{x}^{(1)}$	0	2	3	2/3	2	1
$\mathbf{x}^{(2)}$	1	0	1	0/1	2	1
$\mathbf{x}^{(3)}$	2	0	3	0/3	2	3

- Wrap everything up:
  1. For class j vs. k, transform all $(\mathbf{x}^{(i)}, \mathbf{c}^{(i)})$ to $(\mathbf{x}^{(i)}, \arg\min_{l \in \{j,k\}} \mathbf{c}^{(i)}[l])$ with sample-wise weight $|\mathbf{c}^{(i)}[j] - \mathbf{c}^{(i)}[k]|$.
  2. Train a weighted binary classifier $f^{(j,k)}$ using the above
  3. Repeat step 1 and 2 for different (j,k).
  4. Predict using the votes from all $f^{(j,k)}$.
- Theoretical guarantee:
  test costs of final classifier $\leq 2 \sum_{j<k}$ test cost of $f^{(j,k)}$.

CSOVO

- Example continued

	$\mathbf{c}^{(i)}[1]$	$\mathbf{c}^{(i)}[2]$	$\mathbf{c}^{(i)}[3]$	$\mathbf{c}^{(i)}[1 \text{ vs } 3]$	$\tilde{y}^{(i)}[1 \text{ vs } 3]$	$w^{(i)}[1 \text{ vs } 3]$
$\mathbf{x}^{(1)}$	0	2	3	0/3	1	3
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