OPTIMA: RANK-DEFICIENT CASE

Example: Assume $\mathbf{A}$ is **not** full rank but has a zero eigenvalue with eigenvector $\boldsymbol{v}_0$.

- Recall: $\boldsymbol{v}_0$ spans null space of $\mathbf{A}$, i.e., $\mathbf{A}(\alpha \boldsymbol{v}_0) = \mathbb{0}$ for each $\alpha \in \mathbb{R}$
- $\implies \mathbf{A}(\mathbf{x} + \alpha \boldsymbol{v}_0) = \mathbf{A}\mathbf{x}$
- Since $\nabla q(\mathbf{x}) = 2\mathbf{A}\mathbf{x} + \boldsymbol{b}$:

$$\nabla q(\mathbf{x} + \alpha \boldsymbol{v}_0) = 2\mathbf{A}(\mathbf{x} + \alpha \boldsymbol{v}_0) + \boldsymbol{b} = 2\mathbf{A}\mathbf{x} + \boldsymbol{b} = \nabla q(\mathbf{x})$$

- $\implies$ q has infinitely many stationary points along line $\mathbf{x}^* + \alpha \boldsymbol{v}_0$
- Since $\mathbf{H} = 2\mathbf{A}$, kind of stationary point not changing along $\boldsymbol{v}_0$