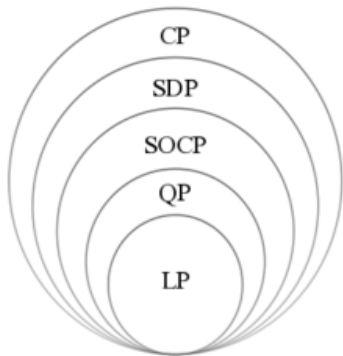
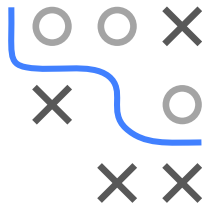


Optimization in Machine Learning

Constrained Optimization

Algorithms for linear programs



Learning goals

- Understand the Simplex algorithm for solving LPs
- Know the two-phase approach for finding a starting point
- Understand how Simplex traverses polytope corners

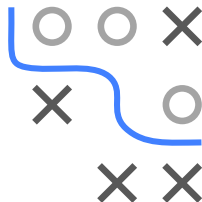
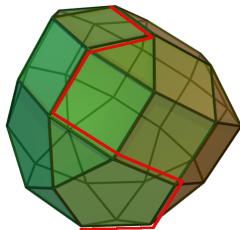
SIMPLEX ALGORITHM

- Most important method for solving linear programs
- Published in 1947 by Georg Dantzig

Basic idea: Start from an arbitrary corner of the polytope.
Run along edges as long as the solution improves. Find a new edge, ...

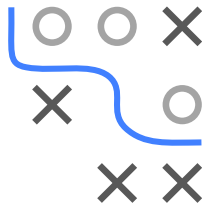
Output: A path along corners of the polytope ending at the optimum

- Since LP is a **convex** optimization problem, the optimal corner found is also a global optimum

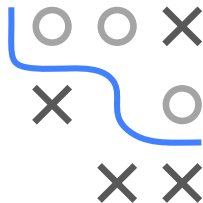


SIMPLEX ALGORITHM

- The Simplex algorithm consists of two phases:
 - **Phase I:** Determination of a **starting point**
 - **Phase II:** Determination of the **optimal solution**
- In **Phase I**, a feasible corner $\mathbf{x}_0$ must be found first
- In **Phase II**, this solution is iteratively improved by searching for an edge that improves the objective and running along it to the next corner



SIMPLEX – EXAMPLE



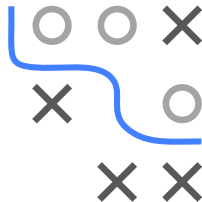
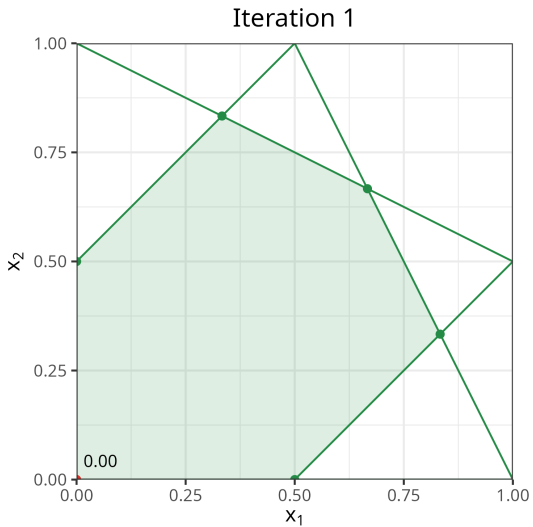
- Consider the following LP:

$$\min_{\mathbf{x} \in \mathbb{R}^2} -x_1 - x_2$$

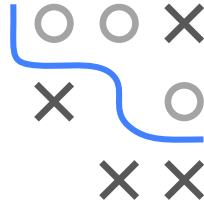
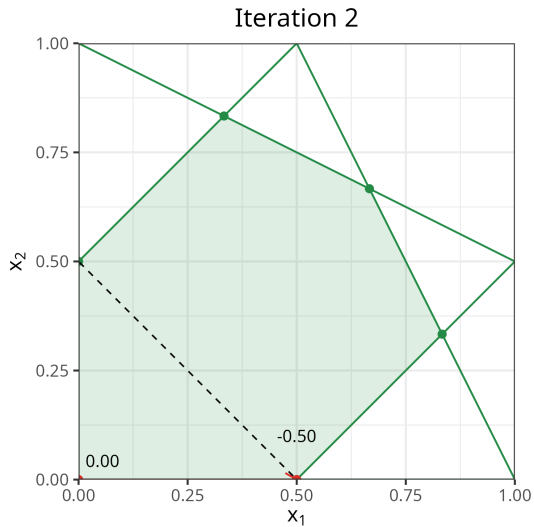
$$\text{s.t. } x_1 - x_2 \geq -0.5, -x_1 - 2x_2 \geq -2, -2x_1 - x_2 \geq -2, -x_1 + x_2 \geq -0.5, \mathbf{x} \geq 0$$

- Starting point is the corner **(0, 0)**

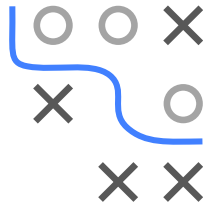
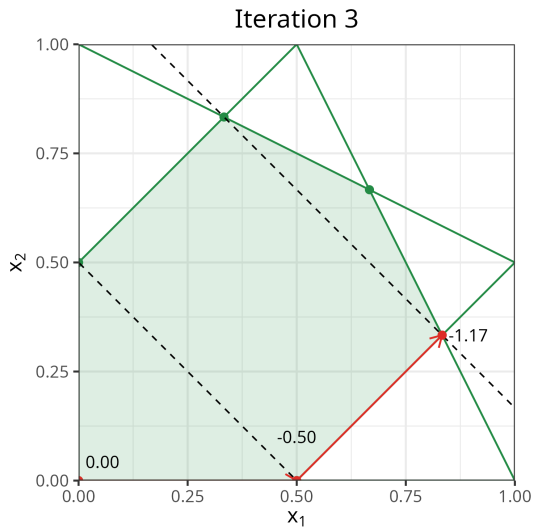
SIMPLEX – EXAMPLE



SIMPLEX – EXAMPLE



SIMPLEX – EXAMPLE



SIMPLEX – EXAMPLE

