

# OPTIMA: RANK-DEFICIENT CASE

**Example:** Assume  $\mathbf{A}$  is not full rank but has a zero eigenvalue with eigenvector  $\mathbf{v}_0$ .

- Recall:  $\mathbf{v}_0$  spans null space of  $\mathbf{A}$ , i.e.,  $\mathbf{A}(\alpha\mathbf{v}_0) = \mathbf{0}$  for each  $\alpha \in \mathbb{R}$
- $\implies \mathbf{A}(\mathbf{x} + \alpha\mathbf{v}_0) = \mathbf{A}\mathbf{x}$
- Since  $\nabla q(\mathbf{x}) = 2\mathbf{A}\mathbf{x} + \mathbf{b}$ :

$$\nabla q(\mathbf{x} + \alpha\mathbf{v}_0) = 2\mathbf{A}(\mathbf{x} + \alpha\mathbf{v}_0) + \mathbf{b} = 2\mathbf{A}\mathbf{x} + \mathbf{b} = \nabla q(\mathbf{x})$$

- $\implies q$  has infinitely many stationary points along line  $\mathbf{x}^* + \alpha\mathbf{v}_0$
- Since  $\mathbf{H} = 2\mathbf{A}$ , kind of stationary point not changing along  $\mathbf{v}_0$

