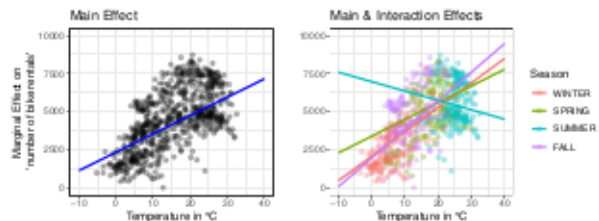
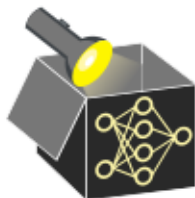


EXAMPLE: INTERACTION EFFECT

Example: Interaction between temp and season will affect marginal effect of temp

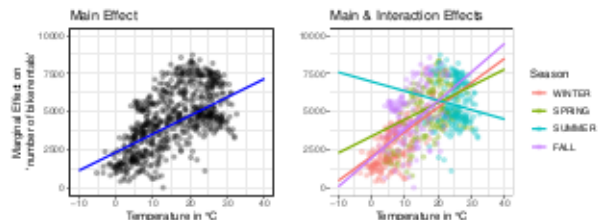


	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

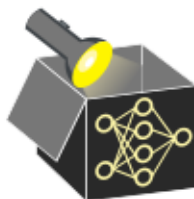


EXAMPLE: INTERACTION EFFECT

Example: Interaction between **temp** and **season** will affect marginal effect of **temp**



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

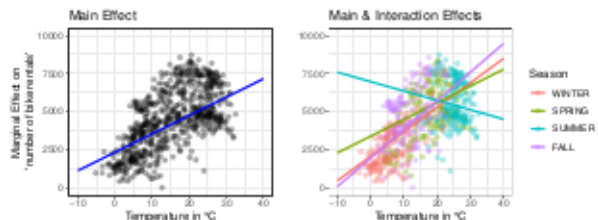


Interpretation: If **temp** increases by 1 °C, bike rentals

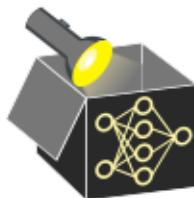
- increase by 160.5 in **WINTER** (reference)

EXAMPLE: INTERACTION EFFECT

Example: Interaction between **temp** and **season** will affect marginal effect of **temp**



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

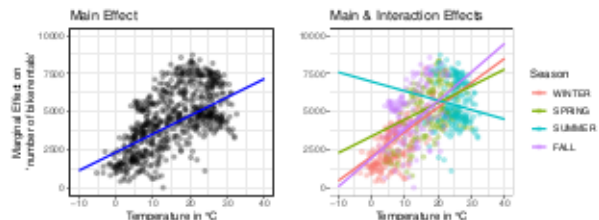


Interpretation: If **temp** increases by 1 °C, bike rentals

- increase by 160.5 in **WINTER** (reference)
- increase by 109.8 (= 160.5 - 50.7) in **SPRING**

EXAMPLE: INTERACTION EFFECT

Example: Interaction between **temp** and **season** will affect marginal effect of **temp**



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

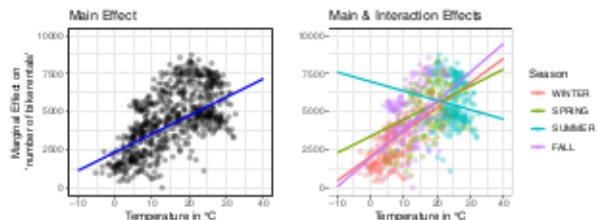


Interpretation: If **temp** increases by 1 °C, bike rentals

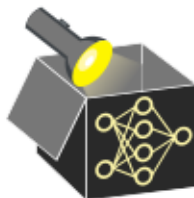
- increase by 160.5 in **WINTER** (reference)
- increase by 109.8 (= 160.5 - 50.7) in **SPRING**
- decrease by -61.5 (= 160.5 - 222) in **SUMMER**

EXAMPLE: INTERACTION EFFECT

Example: Interaction between **temp** and **season** will affect marginal effect of **temp**



	Weights
(Intercept)	3453.9
seasonSPRING	1317.0
seasonSUMMER	4894.1
seasonFALL	-114.2
temp	160.5
hum	-37.6
windspeed	-61.9
days_since_2011	4.9
seasonSPRING:temp	-50.7
seasonSUMMER:temp	-222.0
seasonFALL:temp	27.2

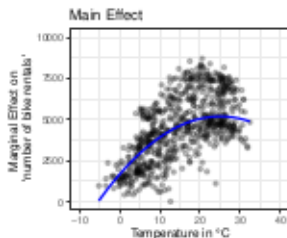


Interpretation: If **temp** increases by 1 °C, bike rentals

- increase by 160.5 in **WINTER** (reference)
- increase by 109.8 (= 160.5 - 50.7) in **SPRING**
- decrease by -61.5 (= 160.5 - 222) in **SUMMER**
- increase by 187.7 (= 160.5 + 27.2) in **FALL**

EXAMPLE: QUADRATIC EFFECT

Example: Adding quadratic effect for `temp`



Interpretation: Not linear anymore!

- `temp` depends on two weights:

$$280.2 \cdot x_{temp} - 5.6 \cdot x_{temp}^2$$

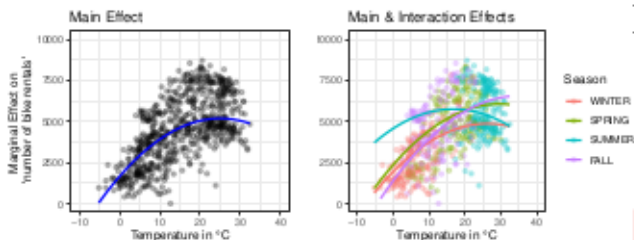
	Weights
(Intercept)	3094.1
seasonSPRING	619.2
seasonSUMMER	284.6
seasonFALL	123.1
hum	-36.4
windspeed	-65.7
days_since_2011	4.7
temp	280.2
temp ²	-5.6



EXAMPLE: QUADRATIC EFFECT



Example: Adding quadratic effect for `temp` (left) and interaction with `season` (right)



	Weights
(Intercept)	3802.1
seasonSPRING	-1345.1
seasonSUMMER	-6006.3
seasonFALL	-681.4
hum	-38.9
windspeed	-64.1
days_since_2011	4.8
temp	39.1
temp ²	8.6
seasonSPRING:temp	407.4
seasonSPRING:temp ²	-18.7
seasonSUMMER:temp	801.1
seasonSUMMER:temp ²	-27.2
seasonFALL:temp	217.4
seasonFALL:temp ²	-11.3

Interpretation: Not linear anymore!

- `temp` depends on multiple weights due to `season`:

$$\rightsquigarrow \text{WINTER: } 39.1 \cdot x_{temp} + 8.6 \cdot x_{temp}^2$$

$$\rightsquigarrow \text{SPRING: } (39.1 + 407.4) \cdot x_{temp} + ((8.6 - 18.7)) \cdot x_{temp}^2$$

$$\rightsquigarrow \text{SUMMER: } (39.1 + 801.1) \cdot x_{temp} + (8.6 - 27.2) \cdot x_{temp}^2$$

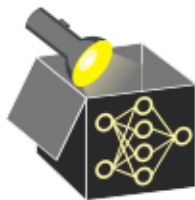
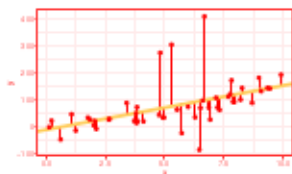
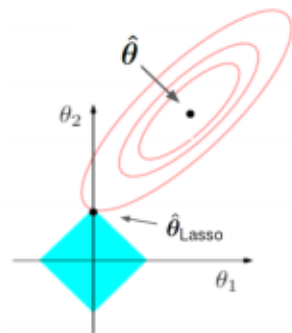
$$\rightsquigarrow \text{FALL: } (39.1 + 217.4) \cdot x_{temp} + (8.6 - 11.3) \cdot x_{temp}^2$$

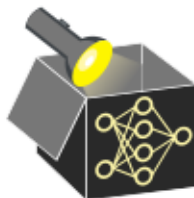
$$\rightsquigarrow \text{FALL: } (39.1 + 217.4) \cdot x_{temp} + (8.6 - 11.3) \cdot x_{temp}^2$$

REGULARIZATION VIA LASSO ► Tibshirani (1996)

- LASSO adds an L_1 -norm penalization term ($\lambda\|\theta\|_1$) to least squares optimization problem
 - ↪ Shrinks some feature weights to zero (feature selection)
 - ↪ Sparser models (fewer features): more interpretable
- Penalization parameter λ must be chosen (e.g., by CV)

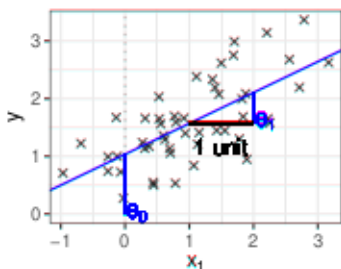
$$\min_{\theta} \left(\underbrace{\frac{1}{n} \sum_{i=1}^n (y^{(i)} - \mathbf{x}^{(i)\top} \theta)^2}_{\text{Least square estimate for LM}} + \lambda \|\theta\|_1 \right)$$





Example (interpretation of weights analogous to LM):

- LASSO with main effects and interaction **temp with season**
- λ is chosen \rightsquigarrow 6 selected features ($\neq 0$)
- **LASSO shrinks weights of single categories** separately (due to dummy encoding)
 \rightsquigarrow No feature selection of whole categorical features (only w.r.t. category levels)
 \rightsquigarrow **Solution: group LASSO** ► Yuan and Lin (2006)



	Weights
(Intercept)	3135.2
seasonSPRING	767.4
seasonSUMMER	0.0
seasonFALL	0.0
temp	116.7
hum	-28.9
windspeed	-50.5
days_since_2011	4.8
seasonSPRING:temp	0.0
seasonSUMMER:temp	0.0
seasonFALL:temp	30.2