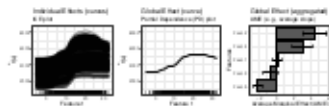


# Interpretable Machine Learning

## Individual Conditional Expectation (ICE) Plot



### Learning goals

- ICE curves as local effect method
- How to sample grid points for ICE curves

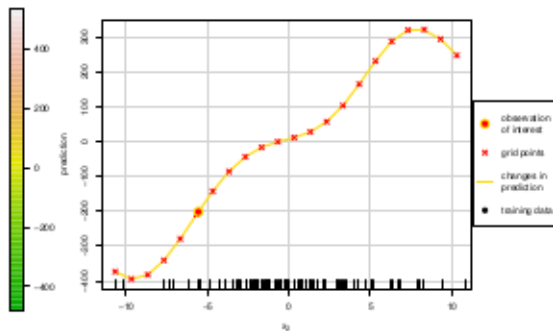
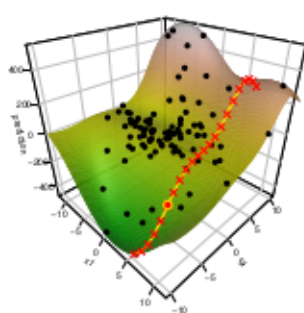
# MOTIVATION

**Question:** How does changing values of a single feature of an observation affect model prediction?

**Idea:** Change values of observation and feature of interest, and visualize how prediction changes

**Example:** Prediction surface of a model (left), select observation and visualize changes in prediction for different values of  $x_2$  while keeping  $x_1$  fixed

⇒ **local interpretation**



# INDIVIDUAL CONDITIONAL EXPECTATION (ICE)

Goldstein et al (2013)



Partition each observation  $\mathbf{x}$  into  $\mathbf{x}_S$  (feature(s) of interest) and  $\mathbf{x}_{-S}$  (remaining features)

↪ In practice,  $\mathbf{x}_S$  consists of one or two features (i.e.,  $|S| \leq 2$  and  $-S = S^c$ ).

Formal definition of ICE curves:

- Choose grid points  $\mathbf{x}_S^* = \mathbf{x}_S^{*(1)}, \dots, \mathbf{x}_S^{*(g)}$  to vary  $\mathbf{x}_S$
- Plot point pairs  $\left\{ \left( \mathbf{x}_S^{*(k)}, \hat{f}_{S,ICE}^{(i)}(\mathbf{x}_S^{*(k)}) \right) \right\}_{k=1}^g$   
where  $\hat{f}_{S,ICE}^{(i)}(\mathbf{x}_S^*) = \hat{f}(\mathbf{x}_S^*, \mathbf{x}_{-S}^*)$
- For each  $k$  connect point pairs to obtain **ICE curve**

↪ ICE curves visualize how prediction of  $i$ -th observation changes after varying its feature values indexed by  $S$  using grid points  $\mathbf{x}_S^*$  while keeping all values in  $-S$  fixed

	$\mathbf{x}_S$		$\mathbf{x}_{-S}$
$i$	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$
1	1	4	7
2	2	5	8
3	3	6	9

# ICE CURVES - ILLUSTRATION



## 1. Step - Grid points:

Sample grid values  $\mathbf{x}_S^{(1)}$ ,  $\mathbf{x}_S^{(2)}$ ,  $\mathbf{x}_S^{(3)}$  along feature of interest  $\mathbf{x}_S$  and replace vector  $\mathbf{x}^{(i)}$  in data with grid

⇒ Create new artificial points for  $i$ -th observation (here:  $\mathbf{x}_S^* = x_1^* \in \{1, 2, 3\}$  scalar)

$x_S$	$x_2$	$x_3$
1	4	7

$x_S$	$x_2$	$x_3$
2	4	7

$x_S$	$x_2$	$x_3$
3	4	7

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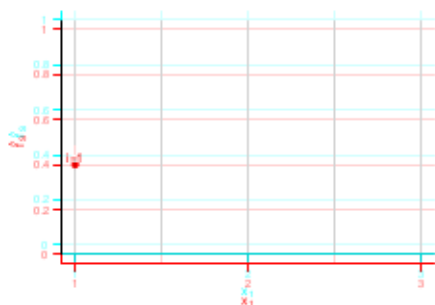


$x_3$	$x_2$	$x_1$	$f$
1	4	7	0.4

$i$	$x_3$	$x_2$	$x_1$
1	1	4	7
2	2	5	8
3	3	6	9

$x_3$	$x_2$	$x_1$	$f$
2	4	7	0.6

$x_3$	$x_2$	$x_1$	$f$
3	4	7	0.7



## 2. Step - Predict and visualize:

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For each artificially created data point of  $i$ -th observation, plot prediction  $\hat{f}_{S,ICE}^{(i)}(\mathbf{x}_S^*)$  vs. grid values  $\mathbf{x}_S^*$ .

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$$\hat{f}_{1,ICE}^{(1)}(\mathbf{x}_1^*) = \hat{f}(\mathbf{x}_1^*, \mathbf{x}_{2,3}^{(1)}) \text{ vs. } \mathbf{x}_1^* \in \{1, 2, 3\}$$
$$\hat{f}_{1,ICE}^{(1)}(\mathbf{x}_1) = \hat{f}(\mathbf{x}_1, \mathbf{x}_{2,3}^{(1)}) \text{ vs. } \mathbf{x}_1 \in \{1, 2, 3\}$$

# ICE CURVES - ILLUSTRATION



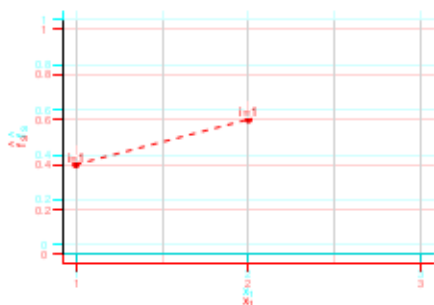
	$x_1$	$x_2$	$x_3$	$f$
1	1	4	7	0.4
2	2	5	8	0.6
3	3	6	9	0.7

$x_1$	$x_2$	$x_3$	$f$
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2	4	7	0.6
3	4	7	0.7

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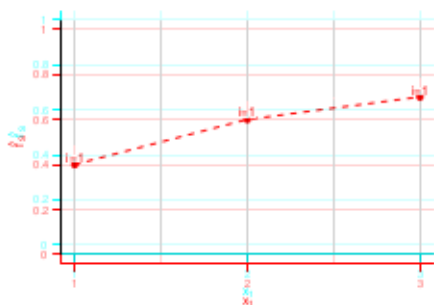
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$$\hat{f}_{1,ICE}^{(i)}(\mathbf{x}_S^*) = \hat{f}(\mathbf{x}_1^*, \mathbf{x}_{2,3}^{(i)}) \text{ vs. } \mathbf{x}_S^* \in \{1, 2, 3\}$$

# ICE CURVES - ILLUSTRATION



$x_1$	$x_2$	$x_3$	$f$
1	4	7	0.4



$i$	$x_1$	$x_2$	$x_3$
1	1	4	7
2	2	5	8
3	3	6	9

$x_1$	$x_2$	$x_3$	$f$
2	4	7	0.6

$x_1$	$x_2$	$x_3$	$f$
3	4	7	0.7

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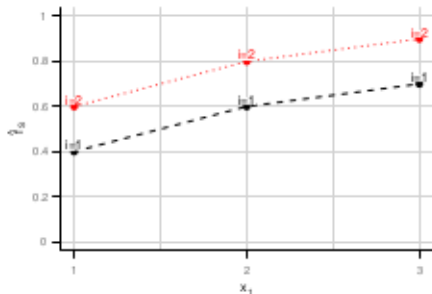
$$\hat{f}_{1,ICE}^{(1)}(x_1^*) = \hat{f}(x_1^*, \mathbf{x}_{2,3}^{(1)}) \text{ vs. } x_1^* \in \{1, 2, 3\}$$
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# ICE CURVES - ILLUSTRATION

$i$	$x_1$	$x_2$	$x_3$	$y_i$
1	1	4	7	0.4
2	2	5	8	0.8
3	3	6	9	0.7

$x_1$	$x_2$	$x_3$	$f_i$
1	4	7	0.4
1	5	8	0.6
2	4	7	0.6
2	5	8	0.8
3	4	7	0.7
3	5	8	0.9



### 3. Step - Repeat for other observations:

### 3. Step - Repeat for other observations:

ICE curve for  $i = 2$  connects all predictions at grid values associated to  $i$ -th observation.

observation.



# ICE CURVES - ILLUSTRATION



$i$	$x_1$	$x_2$	$x_3$
1	1	4	7
2	2	5	8
3	3	6	9

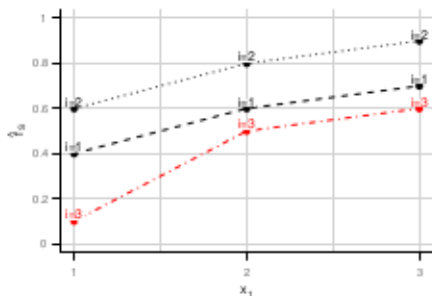
$x_1$	$x_2$	$x_3$	$f_i$
1	4	7	0.4
1	5	8	0.6
1	6	9	0.1

$x_1$	$x_2$	$x_3$	$f_i$
2	4	7	0.6
2	5	8	0.8
2	6	9	0.5

$x_1$	$x_2$	$x_3$	$f_i$
3	4	7	0.7
3	5	8	0.9
3	6	9	0.6



**3. Step - Repeat for other observations:**

**3. Step - Repeat for other observations:**

ICE curve for  $i = 3$  connects all predictions at grid values associated to  $i$ -th observation.

observation.

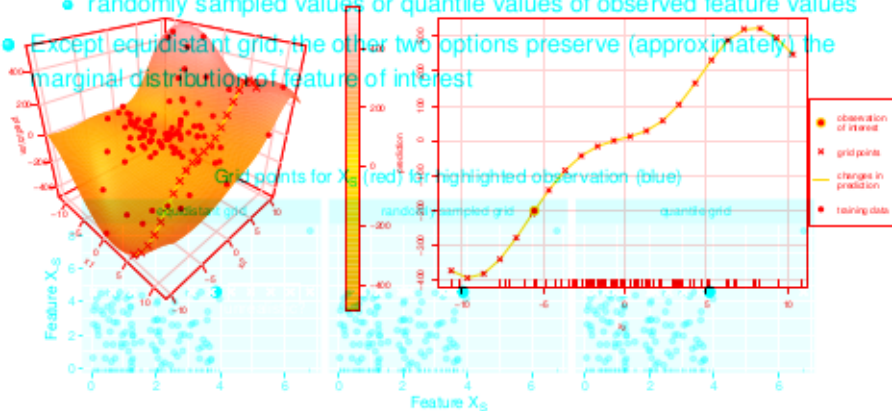
# ICE CURVES INTERPRETATION

**Example:** Prediction surface of a model (left), select observation and visualize changes in prediction for different values of  $x_2$  while keeping  $x_1$  fixed

Common choices for grid values are  
⇒ **local interpretation**

- equidistant grid values within feature range
- randomly sampled values or quantile values of observed feature values

Except equidistant grid, the other two options preserve (approximately) the marginal distribution of feature of interest



# COMMENTS ON GRID VALUES

- Plotting ICE curves involves generating grid values  $\mathbf{x}_G^*$ ; visualized on x-axis
- Common choices for grid values are
  - equidistant grid values within feature range
  - randomly sampled values or quantile values of observed feature values
- Except equidistant grid, the other two options preserve (approximately) the marginal distribution of feature of interest
- Correlations/interactions  $\rightsquigarrow$  unrealistic values in all three methods

