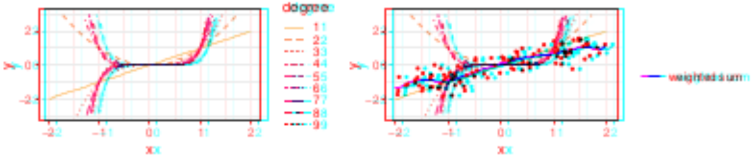
POLYNOMIAL REGRESSION

- Simple & flexible choice for basis funs: *d*-**polynomials**
- Idea: map x_j to (weighted) sum of its monomials up to order $d \in \mathbb{N}$

$$\phi^{(d)} : \mathbb{R} \to \mathbb{R}, \quad x_j \mapsto \sum_{k=1}^{d} \beta_k x_j^k$$



- How to estimate coefficients β_k?
  - Both LM & polynomials **linear** in their params $\rightsquigarrow$ merge
  - E.g., $f(\mathbf{x}) = \theta_0 + \theta_1 \phi^{(d)}(x) = \theta_0 + \sum_{k=1}^{d} \theta_{1,k} x^k$

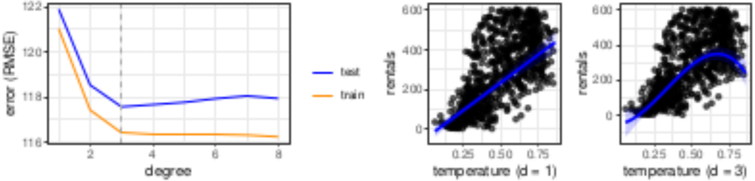
$$\rightsquigarrow \mathbf{X} = \begin{pmatrix} 1 & x^{(1)} & (x^{(1)})^2 & \dots & (x^{(1)})^d \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x^{(n)} & (x^{(n)})^2 & \dots & (x^{(n)})^d \end{pmatrix}, \quad \boldsymbol{\theta} \in \mathbb{R}^{d+1}$$

BIKE RENTAL EXAMPLE

- OpenML task `dailybike`: predict `rentals` from weather conditions
- Hunch: non-linear effect of `temperature` $\leadsto$ include with polynomial:

$$f(\mathbf{x}) = \sum_{k=1}^{d} \theta_{\text{temperature},k} \, x_{\text{temperature}}^{k} + \theta_{\text{season}} x_{\text{season}} + \theta_{\text{humidity}} x_{\text{humidity}}$$

- Test error[2] confirms suspicion $\leadsto$ minimal for $d = 3$



d	θ_0	$\theta_{\text{temp},1}$	$\theta_{\text{temp},2}$	$\theta_{\text{temp},3}$	θ_{sSPRING}	θ_{sSUMMER}	θ_{sFALL}	θ_{hum}	**test error**
1	377.3	2778.2			101.0	57.0	80.1	-230.0	**121.9**
3	419.3	2645.8	-963.1	-430.9	71.9	75.8	56.6	-283.8	**117.6**

- Conclusion: flexible effects can improve fit/performance

[2] Reliable insights about model performance only via separate test dataset not used during training (here computed via 10-fold *cross validation*). Much more on this in Evaluation chapter.