LINEAR CLASSIFIERS

Linear classifiers are an important subclass of classification models. If the discriminant function(s) $f_k(\mathbf{x})$ can be specified as linear function(s) (possibly through a rank-preserving, monotone transformation $g : \mathbb{R} \to \mathbb{R}$), i. e.

$$g(f_k(\mathbf{x})) = \boldsymbol{w}_k^\top \mathbf{x} + b_k,$$

we will call the classifier a **linear classifier**.

NB: $\boldsymbol{w}_k$ and b_k do not directly refer to the parameters $\boldsymbol{\theta}_k$ of k-th scoring function f_k but the transformed version.

LINEAR CLASSIFIERS / 2

We can also easily show that the decision boundary between classes *i* and *j* is a hyperplane. For every **x** where there is a tie in scores:

$$
\begin{aligned}
f_i(\mathbf{x}) &= f_j(\mathbf{x}) \\
g(f_i(\mathbf{x})) &= g(f_j(\mathbf{x})) \\
\mathbf{w}_i^\top \mathbf{x} + b_i &= \mathbf{w}_j^\top \mathbf{x} + b_j \\
(\mathbf{w}_i - \mathbf{w}_j)^\top \mathbf{x} + (b_i - b_j) &= 0
\end{aligned}
$$

This is a **hyperplane** separating two classes.